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A Study of the True Height Analysis Methods

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ABSTRACT

In this paper, a computer routine, POLAN, was used to convert an ionogram recorded at Cleary, Alaska to many electron density true height profiles by specifying various values of MODE, START and VALLEY parameters. These profiles are then compared with that measured by the Chatanika incoherent scatter radar. This comparison allows us to evaluate the difference of the electron density profiles respectively derived by using ionospheric sounding and incoherent backscatter techniques and to investigate the existing true height analysis methods.

1. INTRODUCTION

That part of the atmosphere above about 60 kilometers, where free electrons exist in numbers sufficient to influence the travel of radio waves, is called the ionosphere. Most of our knowledge of the ionosphere comes from remote sensing by radio waves. The conventional equipment for measuring the virtual height of the ionosphere is a sweep-frequency pulsed radar device called an ionosonde. The measurement of the frequency virtual height variation of the ionosphere obtained by an ionosonde is called an ionogram.

An ionogram displays the variation of the virtual height of reflection with frequency, where the virtual height is equivalent to one-half the time-of-flight of the transmitted radio wave times the speed of light c. However, a vertically incident radio wave pulse is affected by the ionospheric refractive index μ and travels upwards with a group velocity $u=\mu c$, until it is reflected at some height h_0, where $\mu=0$, and returns to the ground. The height h_0 is called the ''true height'' (or ''real height'') of the reflection. Note that at ground the true height $h=0$ and the refractive index $\mu=1$. The relation between the virtual height and the true height is given by:

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$$h'(f) = \int_0^{h_0(f)} \mu'(N,B,f)\,dh \tag{1}$$

where h' and f represent the virtual height of reflection and the frequency of radio wave; $\mu' = 1/\mu$ is the group refractive index; N and B are the electron density and geomagnetic field strength at a certain true height h.

Electron density profiles can provide detailed information about the variation of the ionosphere and the effect of those variations on radio wave propagation. From the Appleton-Hartree formula (see Budden, 1985) and the integral equation Eq. (1), it can be seen that the virtual heights displayed in an ionogram are quite different from the true height of the reflecting layer, especially near the layer plasma frequencies. For a more complete understanding of the physics of the ionosphere, it is desirable to obtain the electron density as a function of true height.

Since 1928, a variety of true height methods have been developed to analyze ionograms. Smith (1970) and Wright *et al.* (1988) examined the difference between the electron density profiles converted from ionograms (total reflection sounding technique) and those measured by incoherent scatter technique. However, only a minute fraction of the existing true height analysis methods were employed by Smith and Wright *et al.*.

In this paper an extensive application of POLAN is made. This allows us to investigate the advantages as well as limitations of these methods by evaluating the difference of the electron density true height profiles respectively derived by using ionospheric sounding and incoherent backscatter techniques. In section 2 true height analysis methods are briefly reviewed. In section 3 a general true height analysis program, POLAN, developed by Titheridge (1985) is introduced. In section 4 a comparison between the electron density true height profiles derived from POLAN and that obtained by the incoherent scatter radar (ISR) is presented.

2. METHODS OF TRUE HEIGHT ANALYSIS

The existing true height analysis methods were first summarized in a review paper published by Thomas and Vickers (1959) as a report for the International Geophysical Year. In their review paper, the various methods are classified into the two categories. The classifications are called the model (or comparison) method and the integral equation method. Those methods developed before 1959 were summarized by Thomas and Vickers (1959) and the rest are enriched by Liu (1988). Table 1 adopted from Liu (1988) summaries the existing true height analysis methods.

2.1 Model Method

One of the most important problems in ionospheric research is the determination of the distribution of the electron density as function of height. One may invert the integral

Table 1. Methods of true height analysis (Liu, 1988)

Model Method	**Integral Equation Methods**		
	Direct	Lamination	Polynomial
NO FIELD	NO FIELD	Manual Murray and Hong (1937)	Titheridge (1961, 1967, 1969, 1985)
Parabolic N(h) Appleton and Beynon (1940) Booker and Seaton (1940) Ratcliffe (1951a) Beynon and Thomas (1956)	Appleton (1930) de Groot (1930) Manning (1947) Kelso (1952)	King (1954) Jackson (1956) Titheridge (1975)	Titheridge and Lobb (1977)
Chapman N(h) Pierce (1947)	WITH FIELD	Machine Schmerling (1957)	
Other Distributions N (h) Appleton (1928, 1930) Ratcliffe (1951b)	Rydbeck (1942) Kelso (1954, 1957) Schmerling (1958) Whale (1951)	Thomas, Haselgrove and Robbins (1958) Thomas and Vickers (1959) Titheridge (1979)	
WITH FIELD			
Shin and Whale (1952) Shinn (1953)			

equation in (1) to obtain a solution directly without any prior assumption about the distribution of the electrons with real height. However, since this method generally involves laborious computations, another type of method, which may be called a "comparison method" or "model method" was developed. In the model method, a particular type of electron distribution with true height is assumed, such as the linear, parabolic or Chapman layer profiles (see Liu, 1988) which, when inserted in Eq. (1) gives a model ionogram. By comparing the observed ionogram with a model ionogram, it is possible to decide whether the assumed distribution approximates the actual distribution. If it does, it is an easy matter to determine the fundamental parameters which characterize the actual electron density distribution. Although the method is approximate and does not satisfy all the necessary scientific criteria, it is simple and quick to use, and is therefore convenient for deriving the characteristics (i.e. layer height and layer thickness) of the ionosphere.

2.2 *Integral Equation Method*

In model methods, the theoretical virtual heights are calculated assuming an ideal electron density distribution, however, the actual electron density distribution is still unknown. In the integral equation method, the calculation of electron density distribution is of much greater practical importance, when the virtual heights are given. The integral equation methods may be further subclassified into analytical (direct), lamination and polynomial methods (Thomas and Vickers, 1959; Liu, 1988).

Analytical methods attempt to derive the analytic representation for electron density true height from frequency virtual height in a purely mathematical way. Unfortunately, for most ionograms an exact solution does not exist, unless the virtual height in the ionogram is continuous and varies smoothly. Consequently, it is not always possible to obtain solutions by analytical methods.

After 1959, when the computer came into widespread scientific use, several important advances were made in the techniques which were used to carry out the conversion of a frequency virtual height curve to a electron density true height profile. Two processes which have had success are the lamination methods and polynomial methods. The lamination and polynomial methods have the following advantages:

(a) no *a priori* assumptions are made about the shape of the electron density distribution;

(b) the effect of the Earth's magnetic field is taken into account;

(c) the same adaptive computer codes can be used to analyze ionograms obtained world-wide.

Although the lamination method was presented by Murray and Hoag (1937), it was not widely used until 1954. The term lamination method is used here to denote a general class of methods for inverting the true height integral, using procedures which make explicit use of the assumption that the layer is divided into horizontal strata or laminae, in each of which the refractive index may be described in a relatively simple manner. Lamination methods calculate the electron density profile points step by step, beginning from the lowest frequency. Each step begins from the last salivated real height and is determined to fit the next virtual height (Titheridge, 1988).

Based on the technique of the (simple) linear lamination method, the parabolic and linear offset lamination methods were developed to improve the accuracy of the true height analysis. Liu (1988) indicated that in the case of a simple, smooth ionogram, the results of these methods are accurate, however this is not necessarily true for the case of a complex ionogram.

Alternatively, Titheridge (1961) approached the problem by making the assumption that the electron density distribution in real height may be represented by a polynomial function or a modified polynomial function. Following his initial work, he developed a series of modified polynomial methods to analyze different kinds of ionograms. For smooth ionograms, the simple-polynomial method is adequate, but for complex ionograms, the modified polynomial method is more plausible. There are five types of polynomial methods, termed the simple-polynomial, step-polynomial, parabolic-polynomial, overlapping-polynomial and least-squares-polynomial methods of analysis (for detail see Titheridge, 1985). Titheridge's assumption increases the flexibility of the mathematical techniques and the computer programming, so that the polynomial method can be used to analyze complex ionograms with high accuracy. As a result, the polynomial method has become a powerful tool with which to analyze ionograms.

3. APPLICATIONS OF TRUE HEIGHT ANALYSIS TO DIGITAL IONOSPHERIC SOUNDER DATA

Many computer programs have been developed to derive the electron density real height profile, however, only two of them have been used extensively. These are the **A**utomatic **R**eal **T**ime **I**onogram **S**caler with **T**rue **H**eight (ARTIST) software (Tang *et al.*, 1988) and the **PO**Lynomial **AN**alysis (POLAN) program (Titheridge, 1985). The ARTIST is a routine specially designed for reducing the ionograms obtained from Lowell's digisondes, while the POLAN is a general program for analyzing the ionograms obtained from ionosondes. In this study, the ionogram is recorded by an Dynasonde (Grubb, 1979), therefore, the POLAN program is used.

The POLAN computer program is an automatic, one-pass analysis program, which computes the real height profiles to any required degree, fitting any number of data points. In addition to providing a sequence of experimentally determined frequency virtual height data points, the POLAN computer code requires that the user specify the gyrofrequency, the Earth's magnetic dip angle, the start height of the N(h) profile, the mode (or type) of analysis method and the size of the valley.

As indicated in Titheridge (1985), four basic modes of analysis can be selected. The modes of profile derivation which can be selected are the lamination methods, overlapping polynomial, least squares fit polynomial, or single polynomial methods (see Table 2). The default case is MODE 0, which will adopt a seven-term least squares fit polynomial method (MODE 6).

After setting the MODE parameter, the start height and ionospheric valley should be determined (see Table 3). If START parameter is set to be zero, the start height is

Table 2. Four basic modes of true height analysis invoked by the POLAN Computer Program (Titheridge, 1985)

STANDARD MODES OF ANALYSIS:

MODE 1	– Linear Lamination analysis (with chapman peaks, and valleys).
MODE 2	– Parabolic Lamination analysis, matching end gradients.
MODE 3	– Overlapping Cubics, with no spurious oscillations.
MODE 4	– Fourth Order Overlapping Polynomials.
MODE 5	– Fifth Order Least Squares fit to 6 points (4 virtual + 2 real).
MODE 6	– Sixth Order Least Squares fit to 8 points (5 virtual + 3 real).
MODE 7	– Sixth Order fitting 7 virtual + 3 real heights; calculate 2.
MODE 8	– Sixth Order fitting 8 virtual + 4 real heights; calculate 2.
MODE 9	– Seventh Order fitting 13 virtual + 6 real heights; calculate 3.
MODE 10	A Single Polynomials, fitting 0.73 (nv + 2) terms to nv heights.
MODE 10L,	where L is an integer in the range 3 to 15, uses a single polynomial with L terms to describe each ionospheric layer.
MODE 10L+M	uses L terms for the final layer, and M for the earlier layers.

Table 3. The parameters of the POLAN Computer Program (Titheridge, 1985)

FB negative to use a constant gyrofrequency fh=−fb.
DIP negative to omit physical checks on the calculated profile segments.
START between 0. and 44. defines the plasma frequency for a model start.
start = -1.0 to use a direct start, from the first scaled point.
start < -1.0 for x-starts to use a polynomial from (-start -1.0) mhz.
MODE negative to omit physical relations (c3.3) from start/valley calcons.
VALLEY = 10.0 for a monotonic (no valley) analysis.
valley = 5.0 for a maximum-valley (upper reasonable limit) analysis.
valley = 0.1 to 5.0 multiplies the standard valley by this factor.
valley = -0.1 to -.99 to use -valley as the initial depth, (instead of the default value 0.05 mhz).
valley = -1.0 to iterate both valley depth and width for best fit; (-1.x to iterate from an initial depth of 0.x mhz).
valley = -2.01 to -30 specifies a fixed valley width of 5*int (-valley) km; and any decimal part of valley specifies the in mhz).

determined from a model (Titheridge, 1985 and 1986). When a start height parameter of -1 is specified, a direct start from the first scaled frequency virtual height point is invoked. Normal practice is to set the VALLEY parameter to 0.0 or 1.0 which causes POLAN to use an initial default width of twice the local scale height. The initial default depth is 0.05 MHz. When VALLEY is equal to -1.0, POLAN iterates both valley depth and width for the best fit (Titheridge, 1985).

Finally, the gyrofrequency and dip angle are dependent on the location of the observation. By selecting MODE, START and VALLEY parameters and specifying the gyrofrequency and dip angle, POLAN can be made to analyze typical ionograms.

4. A COMPARISON OF ELECTRON DENSITY PROFILES DERIVING USING IONOSPHERIC SOUNDING AND INCOHERENT BACKSCATTER TECHNIQUES

POLAN incorporates several different analysis methods, which can be invoked by selecting the value of the MODE parameter, and therefore is a useful tool to investigate the advantages and disadvantages of these methods. In 1981, an Advanced Digital Ionosonde (ADI), Dynasonde, (Grubb, 1979) was located at Cleary, Alaska (64.9°*N*, 212.0°*E*) and situated two miles (3 kilometers) away from the Chatanika incoherent scatter radar (ISR), so that the two systems illuminated essentially the same volume of the ionosphere. On October 26, 1981, an ionogram was obtained at 0226:41 UT from the ADI while electron density profile was measured between 0225:03 and 0226:37 UT by the ISR. Figure 1 is the ionogram derived from the ADI, which shows that the ionosphere was quiet and plain-stratified. These provide an excellent opportunity for a comparison of electron density profiles derived using different radar techniques. For a detailed investigation, both qualitative and quantitative comparisons between ADI and ISR results are presented.

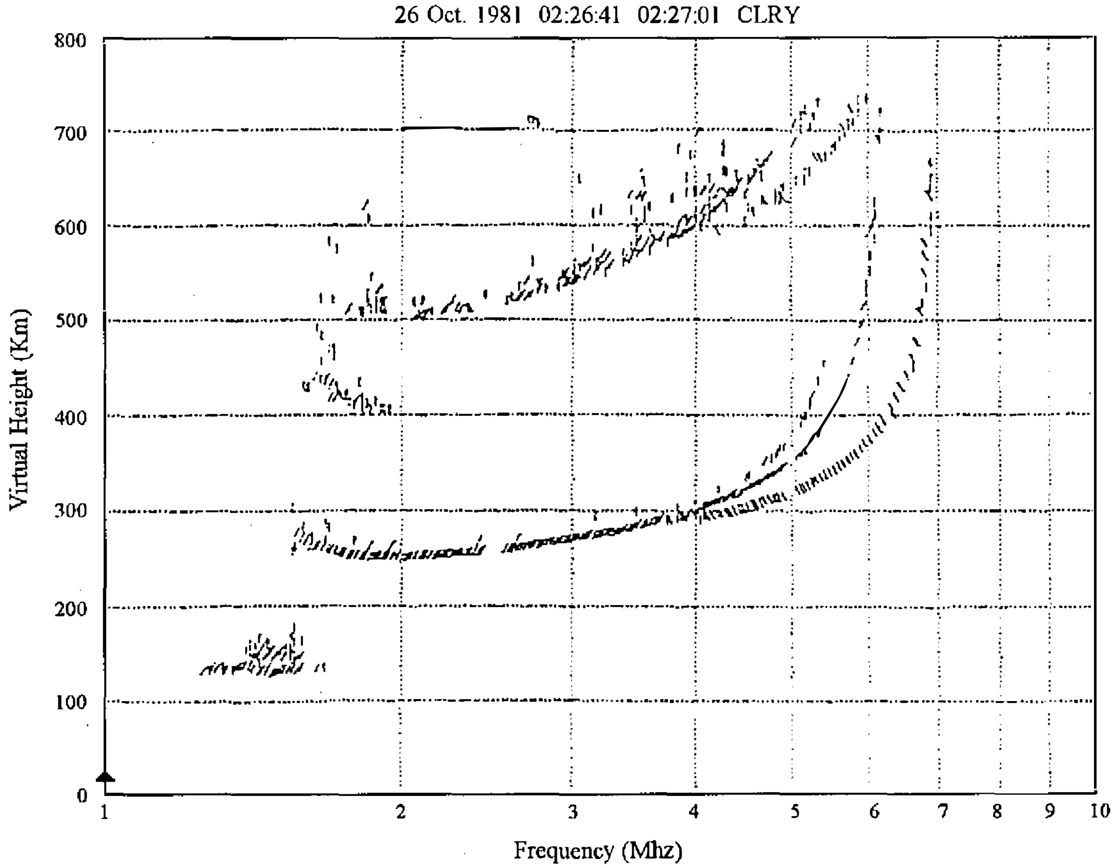

Fig. 1. An ionogram obtained by using an advanced digital ionosonde, Dynasonde, from 0226:41 to 0227:01 UT on October 26, 1981, Cleary, Alaska.

4.1 Qualitative Comparison

To investigate the input data responses to the POLAN, the ADI measurement has been arranged into several subsets to include: (1) F-layer O-ray echoes but not X-ray echoes (O-data); (2) F-layer O- and X-ray echoes but not E-layer echoes (OX-data); and (3) E-layer O-ray echoes and F-layer O- and X-ray echoes (EOX-data). For simplicity, the default value is given in analysis method, start and valley parameters (i.e., MODE 0, START 0 and VALLEY 0).

Figure 2 shows the electron density profiles obtained from the O-, OX- and EOX-data analyzed by using POLAN and recorded by ISR. The solid line in each figure denotes the profile derived by the ISR. It can be seen that: without the E-layer echo, POLAN underestimates the electron density in the E-region; with the E-layer echo, POLAN derives a small E-layer valley and overestimates the electron density below the F-peak. Note when EOX-data used, both ISR and POLAN show a small E-layer valley (or ledge). Therefore, for detailed study the input of POLAN should include both E- and F-

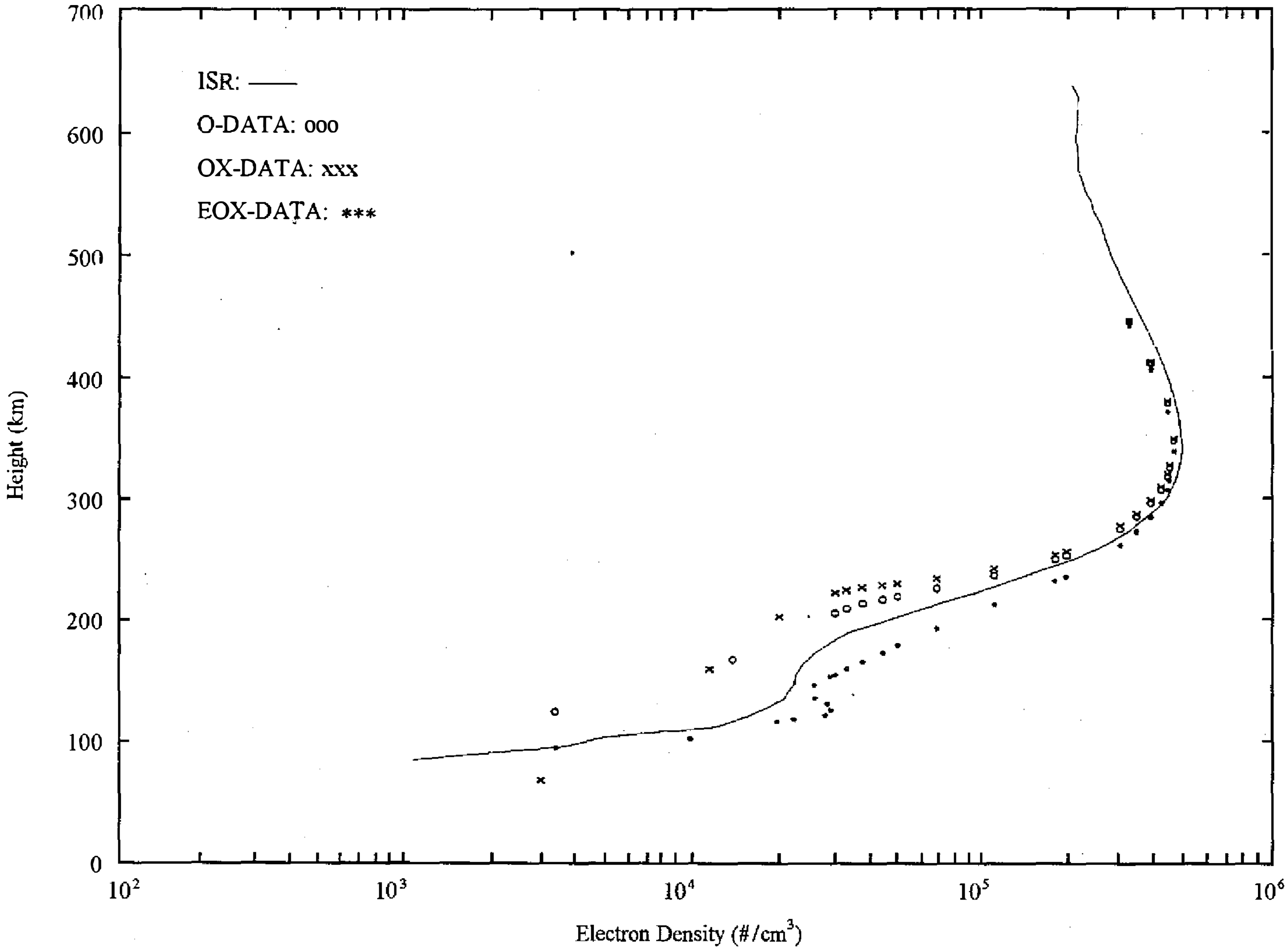

Fig. 2. Comparison of the electron density profiles obtained from ADI and ISR for O-, OX- and EOX data sets used in POLAN.

layer O- and X- ray echoes.

To investigate the effect of different START and VALLEY combinations, the pair (START, VALLEY) has been specified for the combinations (0,0), (0,-1), (-1,0) and (-1,-1). Again, for simplicity reason, the default analysis method is given (i.e. MODE 0). Figure 3 shows that direct start height (START -1) and default valley (VALLEY 0) result in the smallest difference between ADI and ISR measurements.

For a detail study of various analysis methods (MODE), it can be seen from the demonstrations of Figures 2 and 3 that EOX-data should be used; and START -1 and VALLEY 0 are specified. According to the Table 2, MODE has 141 values and, therefore, it is very difficult to investigate all the possible MODE. Hence, the analysis method is generally derived into four groups which are the lamination (MODE 1,2), overlapping polynomial (MODE 3,4), least squares fit polynomial (MODE 5,9,0) and single polynomial (MODE 10,31,95,159) methods. Figures 4(A) to 4(D) show that there is a good agreement between POLAN results and ISR measurements, except when the lamination method (MODE 2) is used.

4.2 Quantitative Comparison

In the quantitative comparison, the differences of the height of the F-layer peak, the electron concentration of the F-layer peak, and the amount of the electron concentration below and above the F-layer peak between POLAN and ISR are calculated.

The differences of the height of the F-layer peak Δh_m is simply written as

$$\Delta h_m = h_{mP} - h_{mI} \qquad (2)$$

where h_{mP} and h_{mI} are the heights of the F-layer peak derived from POLAN and ISR, respectively. Figure 5 shows that the Δh_m usually is less than 3 kilometers except MODE 2,3,141 and 151. Note that the Δh_m at MODE 2 is too big to be plotted (see Figure 5). The differences of the electron concentration of the F-layer peak ΔN_m is expressed as

$$\Delta N_m = N_{mP} - N_{mI} \qquad (3)$$

or

$$\Delta N_m\% = \frac{N_{mP} - N_{mI}}{N_{mI}} \times 100\% \qquad (4)$$

where N_{mP} and N_{mI} are the electron concentration of the F-layer peak derived from POLAN and ISR, respectively. Figure 6 shows that the ΔN_m is between -3.0×10^4 (−6.07%) and -3.1×10^4 #/cm^3 (−6.28%) except MODE 1, 3, and 5. Again, the ΔN_m at MODE 2 is out of plotting scale (see Figure 6). Ideally, POLAN can derive a correct electron density profile up to the peak of the F-layer. Therefore, in addition to the F-peak height and density, the percentage of the electron concentration of the bottomside of the ionosphere is investigated, which can be expressed as

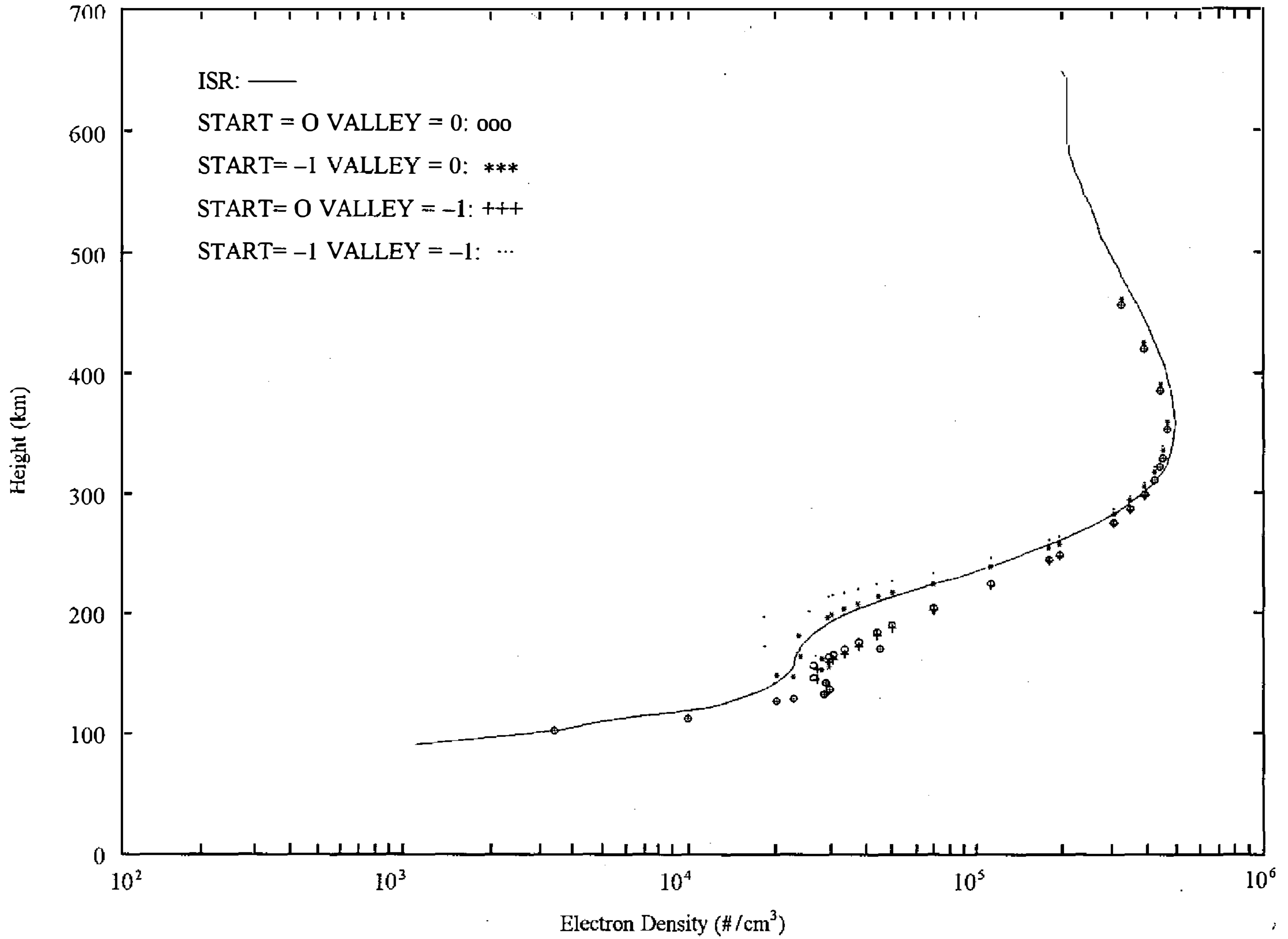

Fig. 3. Comparison of the electron density profiles obtained from ADI and ISR for various values of the START and VALLEY parameters specified with POLAN.

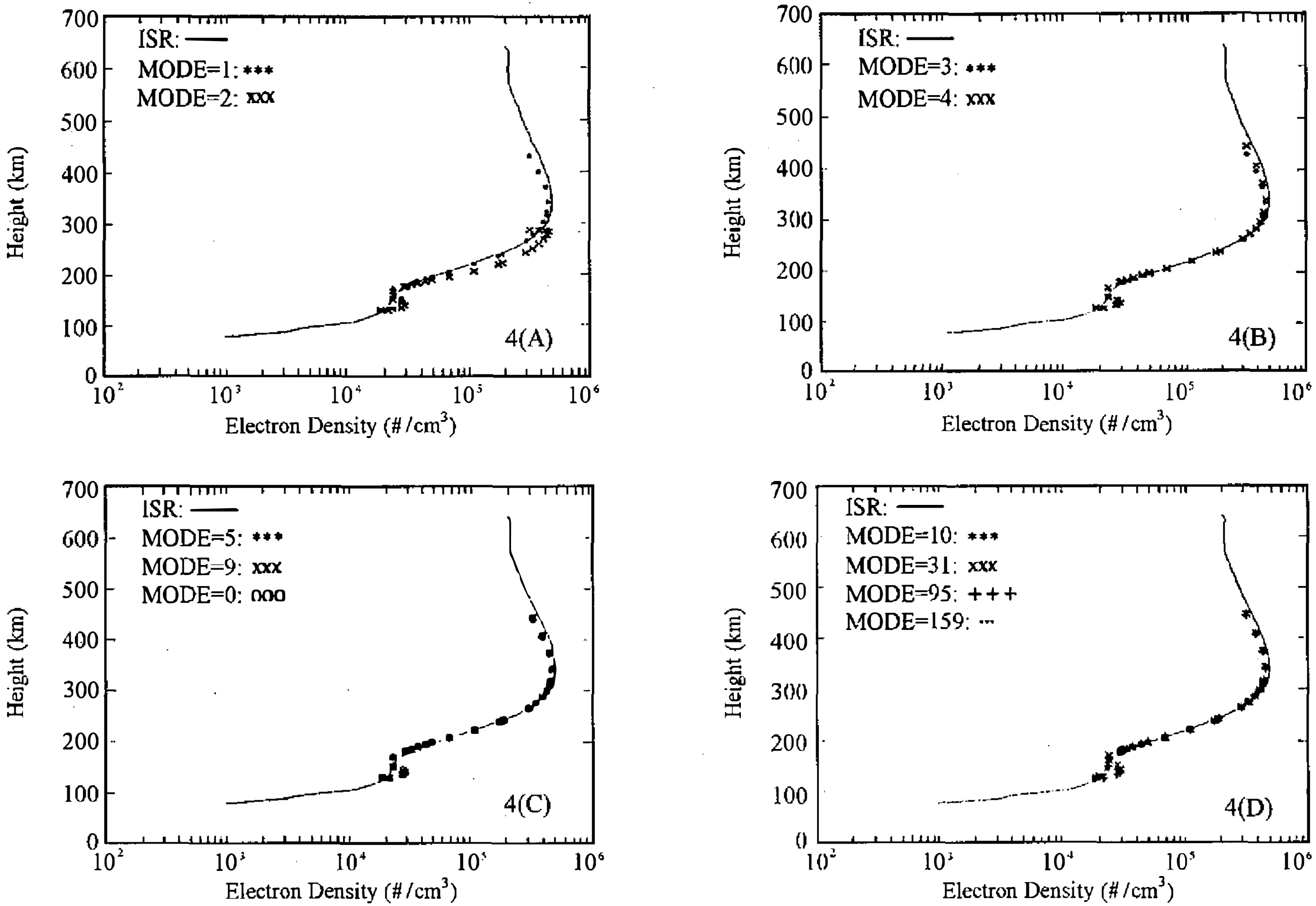

Fig. 4. Comparison of the electron density profiles obtained by ADI and ISR for various true height analysis methods. The EOX-data used, and START -1 and VALLEY 0 specified within POLAN. (A) The lamination methods MODE 1 and 2. (B) The overlapping polynomial methods MODE 3 and 4. (C) The least square fit polynomial methods MODE 5, 9, and 0. (D) The signal polynomial methods MODE 10, 31, 95, and 159.

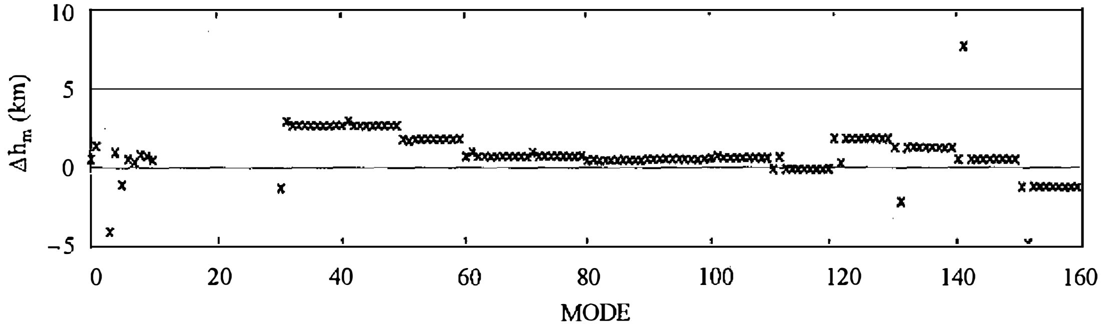

Fig. 5. The difference of the height of the F-region peak obtained from ADI and ISR.

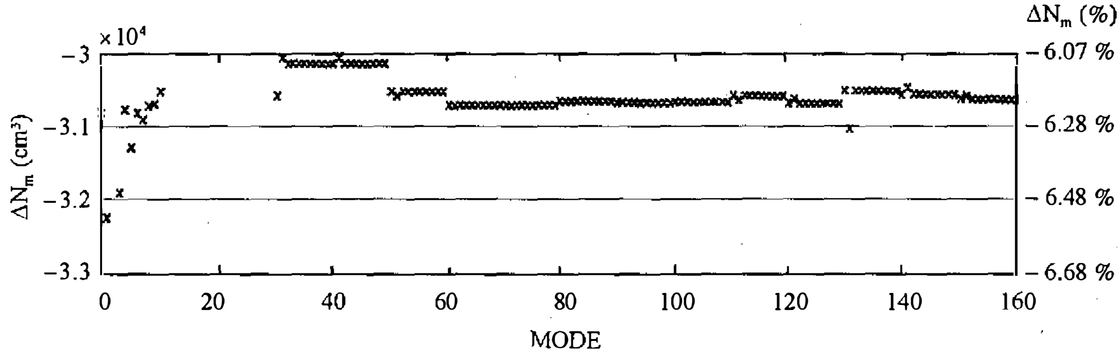

Fig. 6. The difference of the electron density of the F-region peak obtained from ADI and ISR.

$$\Delta N(h)\% = \frac{N_p(h) - N_I(h)}{N_I(h)} \times 100\% \tag{5}$$

where $0 \le h \le h_{mI}$; and N_P and N_I are the electron concentration of POLAN and ISR at a certain height h. Figure 7 is the difference percent of the electron density between POLAN and ISR using Eq. (5). It is apparent that the large differences are resulted from the E-region. Slightly modifying Eq. (5), the percentage of the ionization difference of the bottomside of the ionosphere between these two techniques can be given as

$$\Delta \mathrm{TEC}\% = \frac{\int_0^{h_{mI}} \left| N_p(h) - N_I(h) \right| dh}{\int_0^{h_{mI}} N_I(h)\, dh} \times 100\% \tag{6}$$

The results of Eq. (6) for various MODE are shown in Figure 8. It can be see that the ionization difference is about 1.5%.

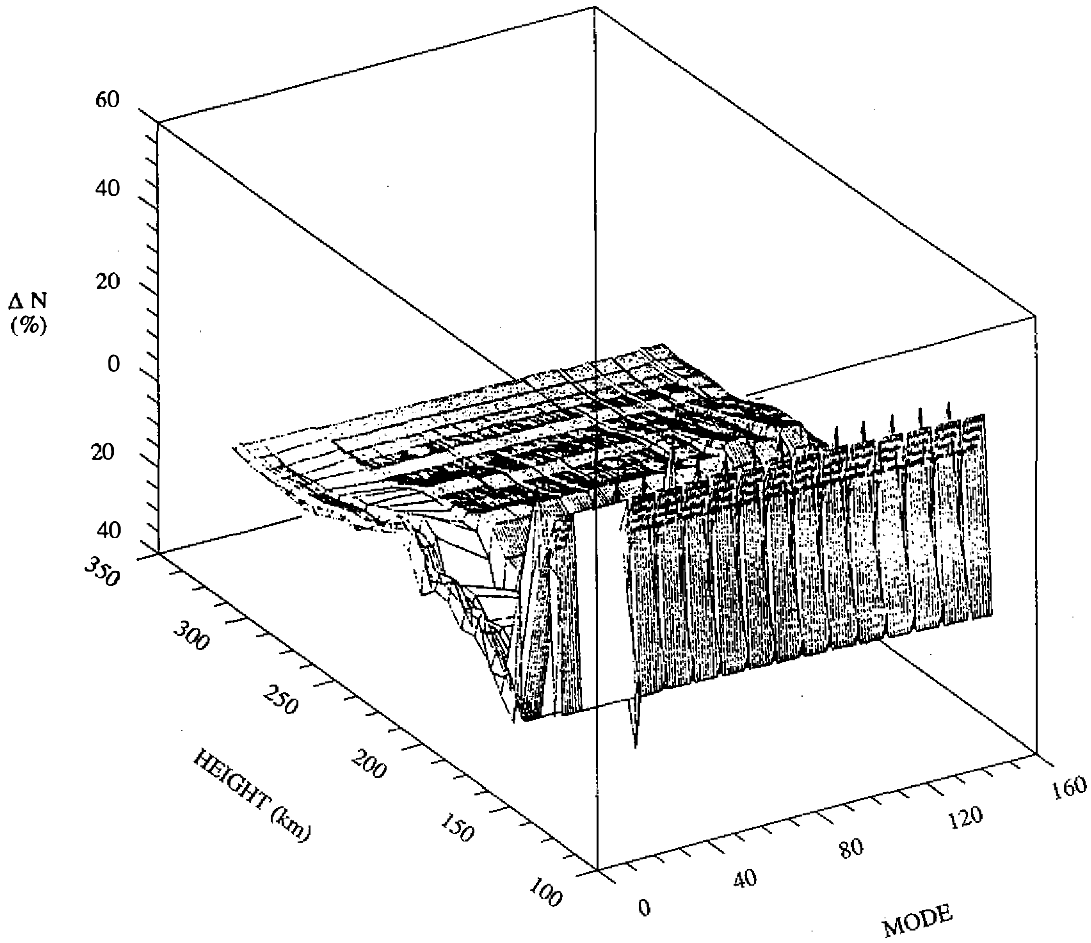

Fig. 7. The difference of the electron density below the F-region peak obtained from ADI and ISR for various MODE.

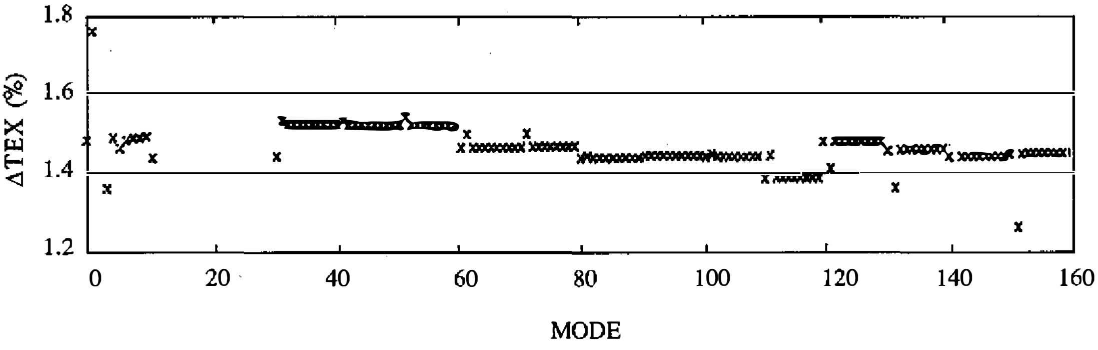

Fig. 8. The difference of the ionization below the F-region peak obtained from ADI and ISR.

5. DISCUSSION

Smith (1970) studying the two layer ionospheric model considered that the difference between the electron density profiles obtained from Arecibo ISR and those derived by a local ionosonde was caused by the valley effect. It is clear, from Figures 2, 3 and 4, that the size and location of the valley is affected by the virtual height data points in the E-region (E-data) and the disagreement below the F-peak between the two profiles is mainly caused by the valley.

Recently, electron density profile obtained from the EISCAT ISR were compared with profiles inverted from the digital ionograms of a colocated ionosonde (Wright *et al.*, 1988). It was found that the agreement between the two profiles depended on the calibration of the ISR. Various calibration (system constant) and pulse length were applied in their experiment, however, the electron density above the F-peak derived from ionosonde by using POLAN was systematically less than that yielded by EISCAT ISR. A similar result also is obtained in this study (see Figures 3 and 4). Note that the F-region data of the Chatanika ISR are fully corrected up to ~ 500 kilometer (Wickwar, private communication). Therefore, we consider that in addition to the calibration and pulse length of ISR, the shape of the F region adapted in the POLAN program may also cause the disagreement above the F-peak between the two sounding techniques.

6. SUMMARY AND CONCLUSION

The survey presented in this paper shows that several methods can be used to derive ionospheric true height profiles. The model method may be used to obtain the ionospheric characteristics (the peak height, scale height and thickness of the layer). For very smooth, monotonically increasing virtual height data either the analytical or the lamination methods can be applied for obtaining electron density profiles. However, a typical electron density real height profile exhibits a valley between the E and F-region and is thus not a monotonically increasing profile. Therefore, the application of the analytical and lamination method are rather limited. Since the diffusion and convection processes in the ionosphere tend to produce an electron density height distribution which varies smoothly. In such cases a smooth and continuous function can be easily represented by a polynomial function and therefore the assumption in the polynomial method is more appropriate.

The qualitative comparisons between the results of POLAN true height analysis and incoherent backscatter technique show that: POLAN applying EOX-data yields the best agreement with the ISR measurement; the direct start and the default valley generally produced the most reasonable results; and POLAN underestimates the electron concentration above the F-peak. The quantitative comparisons between the two results show that the differences of the peak-height, the peak-concentration of the F-region and the ionization below the peak of the F-region are generally less than 3 kilometers, 6.28% and 1.5% with respect to the ISR data.

The disagreement between the two profiles may be mainly attributed by the valley effect. A good agreement between the electron density profiles derived from ISR and HF techniques can be obtained by specifying suitable parameters within POLAN. The lamination methods are inaccurate for true height analysis and the polynomial methods generally yields good agreement with the ISR measurement.

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電離層實高分析方法之研究

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摘要

本文利用電腦實高分析程式POLAN並藉由設定其參數：MODE, START和VALLEY將在Cleary, Alaska所測得的電離圖轉換爲電子濃度之實高分布。然後，將此電子濃度之實高分布和Chatanika異調散射雷達的觀測資料相互比較。由此比較結果可以得知二者在電子濃度之實高分布上的差異，進而瞭解各種實高分析方法之優劣。