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The Determination of the Mechanisms of Ionospheric ULF Oscillations

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ABSTRACT

Variations of ionospheric total electron content (TEC) and/or oscillations of ionospheric Doppler velocity are frequently observed in association with ultra low frequency (ULF) geomagnetic pulsations. Theoretical and numerical models have been developed to study these ionospheric signatures of pulsations. Extending the previous models and with the application of a linear perturbation method, this study shows that the phase differences between variations of TEC and ULF pulsations in the northward component of the geomagnetic field due to the advection and compression mechanisms are 0° and 180°, respectively. It is also shown that ionospheric Doppler velocity oscillations lag and lead by 90° ULF pulsations of the northward component of the geomagnetic field, also caused by the advection and compression mechanisms. Furthermore, we find that TEC variations tend to lead ionospheric Doppler velocity oscillations by 90°.

(Key words: Ionosphere, Doppler velocity, Geomagnetic pulsation)

1. INTRODUCTION

Many experimental studies have reported a close correlation between ultra low frequency (ULF) geomagnetic pulsations and variations of total electron content (TEC) (Davies and Hartmann, 1976; Okuzawa and Davies, 1981) or in the Doppler velocity oscillations in ionospherically reflected radio waves (Klostermeyer and Röttger, 1976; Menk *et al.*, 1983; Sutcliffe and Poole, 1984; Watermann, 1987). A theoretical analysis of the variations in TEC and geomagnetic pulsations was carried out by Poole and Sutcliffe (1987), who concluded that TEC variations can be attributed to either of two physical mechanisms. Many of the experimental measurements of the phase difference between simultaneous ground-based magnetometer measurements of ULF geomagnetic pulsations and fixed-frequency ionospheric

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measurements of echo Doppler velocity oscillations have been interpreted in terms of the Poole and Sutcliffe (1987) model. For example, Jarvis and Gough (1988) discussed observations which were in agreement with the $\mathbf{E} \times \mathbf{B}$ drift and plasma compression mechanisms developed in the Poole and Sutcliffe model. A recent study by Tedd *et al.* (1989) found that the phase differences between ground-level ULF geomagnetic pulsations and ionospheric Doppler oscillations varied from event to event.

There are two important points which we would like to make note of here, the first being that variations in TEC and ionospheric Doppler velocity are the manifestations of two different physical processes. As a result, the phase relationship in the theory of Poole and Sutcliffe (1987) may not be applicable to the observations of Jarvis and Gough (1988) or Tedd *et al.* (1989). Secondly, Poole and Sutcliffe (1987) used peak-to-peak values of the ionospheric electric field to evaluate TEC variations in the context of their advection mechanism. The peak-to peak technique can be used to estimate the mean value of the amplitude of variations in TEC, but it is not suitable for studying the phase difference between TEC variations and ground-level geomagnetic pulsations, as discussed in Section 3.1.

A detailed model showing the relationship between ground-level ULF geomagnetic pulsations and ionospheric Doppler oscillations has been developed by Poole *et al.* (1988). They suggested four mechanisms with which the association of ionospheric Doppler velocity oscillations with ground-level ULF geomagnetic pulsations can be interpreted. Sutcliffe and Poole (1989) refined the work of Poole *et al.* (1988) by including the effect of the geomagnetic field-aligned electron velocity and electron collisions. The refinement of Sutcliffe and Poole (1989) significantly modified the amplitude of ionospheric Doppler velocity oscillations in the case of the advection and compression mechanisms. Both Poole *et al.* (1988) and Sutcliffe and Poole (1989) neglected the contribution due to photochemical processes relative to the other mechanisms. They computed the amplitude of ionospheric Doppler velocity oscillations and the phase difference between Doppler velocity oscillations and ground-level ULF pulsations in the northward component of the geomagnetic field caused by the magnetic, advection and compression mechanisms. However, a detailed theoretical study of the phase difference between ionospheric Doppler velocity oscillations and ground-level ULF geomagnetic pulsations has not been carried out. Note that it is the phase difference which can be used to identify the causal mechanism for the ionospheric Doppler velocity oscillations.

In this paper, the work of Poole and Sutcliffe (1987) and Poole *et al.* (1988) has been extended to study the phase differences between TEC variations, ionospheric Doppler velocity oscillations and ground-level ULF geomagnetic pulsations. Our study is divided into three parts. In the first part, a linear perturbation method is used to derive the relationship between the ionospheric electric field fluctuations and TEC variations, instead of the peak-to-peak technique used in earlier work. The phase difference between TEC variations and ground measured ULF geomagnetic pulsations is derived theoretically using the relationship between the ionospheric electric field and the geomagnetic field, following methods developed by Hughes (1974). Using techniques applied in the first part, the second part studies the phase relationship between ionospheric Doppler velocity oscillations and ULF geomagnetic pulsations, again for each causal mechanism. In the third part, we investigate the response of TEC and Doppler velocity to ULF geomagnetic pulsations, from which the phase relationship between TEC variations and ionospheric Doppler velocity oscillations is derived. The notation and derivations to be presented here basically follow the work of Poole and Sutcliffe (1987) and Poole *et al.* (1988) and the results obtained herein can be compared with those studies.

Finally, the theoretical solutions developed here are compared with theoretical results derived by Poole and Sutcliffe (1987); with numerical solutions developed by Poole *et al.*

(1988) and Sutcliffe and Poole (1989); and with observational results presented by Davies and Hartmann (1976), Jarvis and Gough (1988), Yumoto *et al.* (1989) and Liu *et al.* (1993).

2. THEORETICAL SOLUTIONS

In the following analysis, the x, y, z and s-axes represent the positive geomagnetic northward, eastward, downward and upward directions, respectively. In examining the ionospheric and geomagnetic perturbations, the notation $\frac{\partial}{\partial t}$=-$j\omega$ and $\frac{\partial}{\partial z}$=-$jk$ is employed. The assumptions made in this study are that the ionosphere is quiet and quasi-plane stratified and that the electron drift velocity arises from the ionospheric electric and geomagnetic fields . Starting with the continuity equation and applying a linear perturbation method, the phase relation between TEC variations and geomagnetic pulsations is derived in section 2.1. In section 2.2, the phase relation between Doppler velocity oscillations and geomagnetic pulsations is obtained and the same relationship is derived in section 2.3 for TEC variations and Doppler velocity oscillations.

2.1 TEC Variations and Geomagnetic Pulsations

TEC variations caused by the photochemical, advection or divergence mechanisms were considered by Poole and Sutcliffe (1987) and the phase relationship between TEC variations and geomagnetic pulsations due to the divergence mechanism was derived. Here, the phase difference caused by the advection mechanism is derived in detail.

The total electron content in the ionosphere is defined by the integral

$$TEC = \int_R^{Sat} N\, ds \tag{1}$$

where N is the electron concentration and the integration is performed over the straight line path from a ground-based receiver (R) to a geostationary satellite (Sat). The temporal rate of change of the TEC is given by

$$\frac{\partial(TEC)}{\partial t} = \int_R^{Sat} \frac{\partial N}{\partial t} ds \tag{2}$$

The rate of change of the electron concentration, $\partial N/\partial t$, in equation (2) can be evaluated from the continuity equation for electrons in the ionosphere (Rishbeth and Garriott, 1969)

$$\frac{\partial N}{\partial t} = Q - L - \nabla \cdot (N\mathbf{v}) = Q - L - \mathbf{v} \cdot \nabla N - N\nabla \cdot \mathbf{v} \tag{3}$$

where Q and L are the production and loss rates of the electrons and v is the electron velocity. The electron velocity is expressed as

$$\mathbf{v} = \mathbf{E} \times \frac{\mathbf{B}}{B^2} \tag{4}$$

where **B** and **E** denote the magnetic and electric fields in the ionosphere. The geomagnetic field **B**, which can be separated into time independent ($\mathbf{B}_0$) and time dependent (**b**) terms, is written

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{b} = (B_{x0} + b_x)\hat{x} + b_y\hat{y} + (B_{z0} + b_z)\hat{z} \tag{5}$$

where $|\mathbf{b}| << |\mathbf{B}_0|$ and $\mathbf{b} = \mathbf{b}_0\, e^{-j\omega t}$ is the time varying component of **B**, with angular frequency ω. Based on the similar assumption, the ionospheric electric field can be expressed

$$\mathbf{E} = \mathbf{E}_0 e^{-j\omega t} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z} \tag{6}$$

Poole and Sutcliffe (1987) expressed TEC in terms of an average value and contributions from three separate mechanisms

$$TEC = (TEC)_0 + (TEC)_{QL} + (TEC)_{ADV} + (TEC)_{DIV} \tag{7}$$

where the subscripts denote the time independent value (0), and the time dependent value due to the photochemical (QL), advection (ADV) and divergence (DIV) mechanisms. Combining equations (2), (3) and (7), the temporal change of TEC for each of the three mechanisms is written as follows

$$\frac{\partial (TEC)_{QL}}{\partial t} = \int_R^{Sat} [Q - L] ds \tag{8}$$

$$\frac{\partial (TEC)_{ADV}}{\partial t} = -\int_R^{Sat} \mathbf{v} \cdot \nabla N \; ds \tag{9}$$

$$\frac{\partial (TEC)_{DIV}}{\partial t} = -\int_R^{Sat} N\nabla \cdot \mathbf{v} \; ds \tag{10}$$

2.1.1 Photochemical mechanism

Poole and Sutcliffe (1987) considered the possibility that the photochemical production and loss of electrons can influence the electron concentration. From this work it was shown that the photochemical process is an unlikely causal mechanism for TEC oscillations at low and middle latitudes. The conditions under which their conclusion applies are those of a quiet, quasi-plane stratified ionosphere. For such conditions, the electron concentration can be written

$$N = N(z,t) = N_0(z) + N_t(z,t) \tag{11}$$

where $N_t(z,t) << N_0(z)$ and $N_0(z)$ and $N_t(z,t)$ represent the mean and time varying components of the electron concentration at a height z.

2.1.2 Advection mechanism

The variations in TEC caused by the advection mechanism are due to changes in N resulting from the bodily transport of electrons. Upon substitution of equation (4) into equation (9), neglecting the contribution due to the time varying portion of the gradient of the electron concentration given in equation (11) and, integrating equation (9) with respect to time, we find that the variation of TEC due to the advection mechanism is

$$TEC_{ADV} = \int_R^{Sat} \frac{cos(I)}{\omega B_0}\frac{\partial N_0}{\partial z} E_y\, j\; ds \tag{12}$$

where I is the geomagnetic dip angle and E_y is the y component of the ionospheric electric field. Note that there is a 90° phase difference between the variations of TEC_{ADV} and the y component of the electric field.

2.1.3 Divergence mechanism

TEC variation caused by changes in N due to the divergence of electrons from a given volume have been termed the 'divergence' mechanism. Poole and Sutcliffe (1987) and Poole *et al.* (1988) show that the divergence mechanism can be expressed in terms of the ionospheric current and the temporal rate of change of the electric field, as well as the temporal rate of change of the geomagnetic induction field (plasma compression) and the gradient effects of the geomagnetic induction field. The terms which specify the ionospheric current and the temporal rate of change of electric field are much smaller than the remaining two terms and can be neglected. Thus, integrating equation (10), and using the derivation in Poole *et al.* (1988) and in Rishbeth and Hanson (1974), we find that variations in TEC due to the divergence mechanism can be expressed

$$TEC_{DIV} \cong \int_R^{Sat} N[(\frac{b_{xI}cos(I) + b_{zI}sin(I)}{B_0}) - j\frac{6f(\Lambda)}{\omega r B_0}E_y]ds \tag{13}$$

where the subscripts xI and zI denote the x and z components of the geomagnetic perturbation field in the ionosphere, respectively. Λ represents geomagnetic dipole latitude, where $f(\Lambda)$ is a positive quantity and r is the geocentric distance. Equation (13) shows that the phase relationships between the variations of TEC caused by the divergence mechanism and the oscillations of the x and z components of the ionospheric geomagnetic field and the y component of the ionospheric electric field are 0°, 0° and 90°, respectively.

In a study of geomagnetic pulsations, Dungey (1963) considered that disturbances of the magnetic field result from two separate modes, one with and the other without a vertical current. Based on this assumption, the derivation presented by Hughes (1974) can be invoked to obtain the relationships between variations of the ionospheric electric field and geomagnetic pulsations at the surface of the Earth which can be written as

$$E_x = j\omega z\; b_{yG} \tag{14}$$

$$\begin{aligned} E_y &= \frac{jk^2c^2}{\omega\alpha} ln[\frac{\mu_0 c^2\sigma_G - j\omega}{\mu_0 c^2 \sigma_G}]b_{xG} = C_1 b_{xG} + C_2\; j\; b_{xG} \\ &= -\frac{k^2c^2}{\omega\alpha}tan^{-1}(\frac{-\omega}{\mu_0 c^2\sigma_G})b_{xG} + \frac{k^2c^2}{2\omega\alpha}ln[(\frac{\omega}{\mu_0 c^2\sigma_G})^2 + 1]\; j\; b_{xG} \end{aligned} \tag{15}$$

$$E_z = \frac{jkc^2}{-j\omega + \mu_0 c^2\sigma_G} b_{xG} \tag{16}$$

where the subscript G denotes the magnetic field at ground-level. $1/\alpha$ is the scale height of the atmosphere. μ_0 and c are the permeability constant and speed of light in free space and k is the wave number of the perturbation. For ULF oscillations, C_2 is greater than C_1 in equation (15). Equations (14), (15) and (16) indicate that the x, y and z components of the ionospheric electric field lag behind the y, x and x components of the geomagnetic induction field at the ground by 90°. Combining equations (12) and (15), we can conclude that variations of TEC caused by the advection mechanism and pulsations of the x component of the magnetic field at ground-level tend to be in phase.

Equation (13) shows the phase relationship between variations of TEC caused by the divergence mechanism and the pulsations of the geomagnetic field in the ionosphere. According to the results of Hughes and Southwood (1976) and Poole and Sutcliffe (1987), the b_{xI} component should be of the same order but oppositely directed to b_{xG} while b_{zI} is of the same order and directed parallel to b_{zG}. Therefore, from equations (13) and (15), we can conclude that the phase difference between variations of TEC due to the divergence mechanism and the pulsations of the x and z components of the magnetic field at ground-level are 180° and 0°, respectively.

2.2 Doppler Velocity Oscillations and Geomagnetic Pulsations

As in their earlier work, four mechanisms were considered by Poole *et al.* (1988) to cause ionospheric Doppler velocity oscillations. The four mechanisms are denoted as V_1 (the magnetic mechanism), V_2 (the advection mechanism), V_3 (the divergence mechanism) and V_4 (the photo-chemical mechanism). Neglecting mechanism V_4, Poole *et al.* (1988) and Sutcliffe and Poole (1989) computed numerical solutions for both the amplitude of Doppler velocity oscillations and the phase difference between Doppler velocity oscillations and geomagnetic pulsations. In this work, the phase difference between Doppler velocity oscillations and geomagnetic pulsations caused by mechanisms 1, 2 and 3 is derived from a theoretical analysis.

The Doppler velocity, that is the temporal rate of change of the phase height, is a function of the geomagnetic field and the ionospheric electron concentration and can be written

$$V^* = \frac{dh}{dt} = \frac{d}{dt}\int_0^{Z_R} n[B_L(z,t),\ B_T(z,t),\ N(z,t)]dz \tag{17}$$

where B_L and B_T are the longitudinal and transverse components of B_0, h is the phase height of the reflection point (Z_R), and n is the phase refractive index. Poole *et al.* (1988) separated the expression for Doppler velocity into four components

$$V^* = V_1 + V_2 + V_3 + V_4 \tag{18}$$

where

$$\begin{aligned} V_1 = & V_{1L} + V_{1T} = \int_0^{Z_R} [\frac{\partial n}{\partial B_L}\frac{\partial B_L}{\partial t} + \frac{\partial n}{\partial B_T}\frac{\partial B_T}{\partial t}]dz \\ & \cong -j\omega \int_0^{Z_R} [\frac{\partial n}{\partial B_L} b_{zI} + \frac{\partial n}{\partial B_T} b_{xI}]dz \end{aligned} \tag{19}$$

$$V_2 = -\int_0^{Z_R}[\frac{\partial n}{\partial N}(\mathbf{v}\cdot\nabla N)]dz \cong \int_0^{Z_R}[\frac{dn}{dz}\frac{cos(I)}{B}E_y]dz \tag{20}$$

$$\begin{aligned} V_3 &= -\int_0^{Z_R}[\frac{\partial n}{\partial N}N(\nabla\cdot\mathbf{v})]dz \cong V_{3c}+V_{3g} \\ &\cong -\int_0^{Z_R}[j\omega\frac{\partial n}{\partial N}\frac{N}{B_0}(b_{xI}cos(I)+b_{zI}sin(I))+\frac{\partial n}{\partial N}\frac{6N}{B_0 r}\frac{f(\Lambda)}{}E_y]dz \end{aligned} \tag{21}$$

$$V_4 = \int_0^{Z_R}[\frac{\partial n}{\partial N}(Q-L)]dz \tag{22}$$

These four mechanisms can be studied in the context of the refractive index of the ionosphere. The phase refractive index is expressed by the Appleton-Hartree formulas and written

$$n^2 = 1 - \frac{2X(1-X)}{2(1-X)-Y_T^2 \pm \sqrt{Y_T^4+4Y_L^2(1-X)^2}} \tag{23}$$

where $X{=}f_N^2/f_R^2$ and $Y{=}f_H/f_R$; and f_R, f_N and f_H represent probing, plasma and gyro frequencies, respectively. the subscript L and T denote the longitudinal and transverse components.

2.2.1 Mechanism 1

Here, we discuss the so-called magnetic mechanism. The presence of the geomagnetic field causes the phase refractive index to have two characteristic values and the radio wave to travel as two separate waves (the ordinary or O-mode and the extraordinary or X-mode). Following the convention of magneto-ionic theory, the + and - signs denote the O- and X-modes. The refractive index for the O- and X-modes is affected by the magnetic field and the longitudinal and transverse components are written as

$$\frac{\partial n_\pm}{\partial Y_L} = C_\pm[\pm\frac{4Y_L(1-X)^2}{\sqrt{Y_T^4+4Y_L^2(1-X)^2}}] \tag{24}$$

$$\frac{\partial n_\pm}{\partial Y_T} = C_\pm[-2Y_T \pm \frac{2Y_T^3}{\sqrt{Y_T^4+4Y_L^2(1-X)^2}}] \tag{25}$$

and

$$C_\pm = \frac{X(1-X)}{n_\pm[2(1-X)-Y_T^2 \pm \sqrt{Y_T^4+4Y_L^2(1-X)^2}]^2} \tag{26}$$

where X takes on values between 0 and 1, as does the term $X(1-X)$. Since $n_\pm$ is positive, $C\pm$ is always positive for either O- or X-modes. When the longitudinal component of the geomagnetic field is important, we can show that Doppler velocity oscillations of O-mode and X-mode waves will always be out of phase by considering equations (19) and (24). Furthermore, the Doppler velocity oscillations for O- and X-mode waves lead and lag behind pulsations of the z component by 90°. In equation (25), the square root term is always larger than Y_T^2. When the transverse component of the geomagnetic field is important, combining equations (19) and (25) shows that the Doppler velocity oscillations of O-mode and X-mode waves are in phase and the Doppler velocity oscillations for both O- and X-mode waves lag behind the pulsations of the x component of the magnetic field in the ionosphere by 90°.

Again, the b_{xI} component should be of the same order and oppositely directed to b_{xG} while b_{zI} is of the same order and directed parallel to b_{zG}. Therefore, due to the pulsations of the longitudinal component of the magnetic field, the Doppler velocity oscillations for O- and X-mode waves lead and lag behind the ground-level z component ionospheric geomagnetic pulsations by 90°, and due to the pulsations of the transverse component of the geomagnetic field, the Doppler velocity oscillations for both O- and X-mode waves lead the geomagnetic x component pulsations by 90°.

2.2.2 Mechanism 2

Now, we consider the advection mechanism. By combining equations (20) and (15), the phase relationship between the pulsations of the x component of the geomagnetic field at the ground and the oscillations of Doppler velocity caused by the advection mechanism can be expressed as

$$V_2 \cong \int_0^{Z_R} \frac{dn}{dz}\frac{cos(I)}{B}\frac{k^2c^2}{2\omega\alpha} ln[(\frac{\omega}{\mu_0 c^2 \sigma_G})^2+1]j\ b_{xG} \qquad (27)$$

Below the peak of the F-region, dn/dz is positive and equation (27) shows that for oscillations of both O- and X-mode waves, the oscillations of Doppler velocity caused by the advection mechanism tend to lag behind pulsations of the x component of the geomagnetic field at ground-level by 90°.

2.2.3 Mechanism 3

Finally, we treat the divergence mechanism. The phase relationships between the pulsations of the geomagnetic field at ground-level and the oscillations of the Doppler velocity caused by the divergence mechanism can be separately considered as plasma compression and gradient mechanisms. From equation (21), the phase relationship caused by the compression mechanism can be expressed as

$$\begin{aligned} V_3 \cong & \int_0^{Z_R} [\omega\frac{\partial n}{\partial N}\frac{N}{B}(cos(I)b_{xG} - sin(I)b_{zG})j]dz \\ & - \int_0^{Z_R} \left\{ \frac{\partial n}{\partial N}\frac{3f(\Lambda)}{rB_0}\frac{k^2c^2}{\omega\alpha} ln[(\frac{\omega}{\mu_0 c^2\sigma_G})^2+1]j\ b_{xG} \right\} dz \end{aligned} \qquad (28)$$

The first term of equation (28) indicates that the oscillations of Doppler velocity caused by the compression mechanism lead the pulsations of the x component of the ground-level geomagnetic field by 90°, while the oscillations of the Doppler velocity lag behind or lead the pulsations of the z component of the ground-level geomagnetic field by 90°. The second term shows that oscillations of Doppler velocity caused by gradient mechanism lag behind the pulsations of the x component of the geomagnetic field by 90°.

2.3 TEC Variations and Doppler Velocity Oscillations

In order to study the phase difference between TEC variations and Doppler velocity oscillations, these two parameters must be represented in the same coordinate system. Along the z-axis, TEC can be expressed

$$TEC = \int_0^{Z_0} N\, dz \tag{29}$$

where Z_0 is the altitude of the satellite or the reflected height of the radio wave and the integration starts at the Earth's surface. The temporal rate of change of the TEC is given by

$$\frac{\partial(TEC)}{\partial t} = \int_0^{Z_0} \frac{\partial N}{\partial t} dz \tag{30}$$

Using Eulers formula, it can be seen that the temporal rate of change of the TEC variations leads TEC variations by 90°.

The numerical results of Poole *et al.* (1988) and Sutcliffe and Poole (1989) show that the contribution of the magnetic mechanism (V_1) is much smaller than that of either the advection (V_2) or divergence (V_3) mechanisms. Therefore, the Doppler velocity can be approximated by

$$V^* \cong \int_0^{Z_0} \frac{\partial n}{\partial N} \frac{\partial N}{\partial t} dz \tag{31}$$

where $\frac{\partial n}{\partial N}$ is negative. By comparing equations (30) and (31), we find that the phase difference between Doppler velocity oscillations and the temporal rate of change of TEC variations is 180°. Consequently, variations of TEC will lead Doppler velocity oscillations by 90°.

3. DISCUSSION

3.1 Phase Relationship Between TEC Variations and Geomagnetic Pulsations

Poole and Sutcliffe (1987) were the first to investigate the difference in phase between TEC variations and geomagnetic pulsations. The advection mechanism, represented in equations (35) and (48) of Poole and Sutcliffe (1987), shows that the phase difference between TEC variations and pulsations of the geomagnetic induction field is 180°. The phase difference between variations of TEC caused by the divergence mechanism and pulsations of the northward component of the geomagnetic induction field at ground-level was also found to be 180°. Our theoretical study shows that the phase differences between variations of TEC caused by the advection and divergence mechanisms and pulsations of the northward component of the geomagnetic induction field at ground-level are 0° and 180°, respectively.

Both Poole and Sutcliffe (1987) and the current study obtain similar results for the divergence mechanism.

In addition to this similarity, we show that variations of the z component of the geomagnetic field and the y component of the ionospheric electric field also contribute to TEC variations when the divergence mechanism is invoked. However, when deriving the phase relationship caused by the advection mechanism, Poole and Sutcliffe (1987) evaluated variations of TEC assuming an ionospheric electric field magnitude of twice the mean amplitude. This peak-to-peak technique, which neglects the fluctuations of the electric field, can only be used to evaluate the mean value of the amplitude of the variation in TEC but not to study the phase difference.

3.2 Phase Relationship Between Doppler Velocity Oscillations and Geomagnetic Pulsations

The theoretical phase relationship between Doppler velocity oscillations and geomagnetic pulsations obtained in section 2.2 can be compared with numerical values given by Poole *et al.* (1988) and Sutcliffe and Poole (1989).

As shown in Table 1, the theoretical and numerical phases of both V_{1L} and V_{1T} are quite dissimilar. However, the difference in the numerical phase of V_{1L} between the O- and X-modes is 182° which agrees with our theoretical difference of 180°. Moreover, the difference of the numerical phase of V_{1T} between the O- and X-modes is 4°, which again agrees with our theoretical difference of 0°. Based on these comparisons, we conclude that Doppler velocity oscillations in the O- and X-modes due to the longitudinal (z) component of the magnetic field in the ionosphere have a tendency to be in anti-phase and the Doppler velocity oscillations of O- and X-modes due to the transverse (x) component of the magnetic field in the ionosphere have a tendency to be in phase.

Table 1. A Comparison Between the Numerical Phase Difference of Poole *et al.* (1988) and Theoretical Phase Differences in the Current Study.

Component	O-Mode Numerical Amplitude	O-Mode Numerical Phase	O-Mode Theoretical Phase	X-Mode Numerical Amplitude	X-Mode Numerical Phase	X-Mode Theoretical Phase
V_{1L}	0.059	18°	90°	0.091	-164°	-90°
V_{1T}	0.037	128°	90°	0.029	124°	90°
$V_1 = V_{1L} + V_{1T}$	0.058	54°	90°	0.104	-180°	–
V_2	0.388	-83°	-90°	0.473	-77°	-90°
V_{3c}	1.186	168°	90°	0.827	168°	90°
V_{3g}	0.029	-97°	-90°	0.022	-80°	-90°
$V_3 = V_{3c} + V_{3g}$	1.183	169°	90°	0.819	170°	90°
$V^* = V_1 + V_2 + V_3$	1.085	-174°	90°	0.863	-159°	90°

Note that in Table 1 all the numerical phase values are referenced to a ground-level pulsation b_{xG} component of amplitude 1 nT and Phase 0 degree.

Assuming that b_{xI}, b_{yI} and b_{zI} are in phase, for O-mode waves, both the theoretical phase of V_{1L} and V_{1T} are 90°. Therefore, the theoretical phase of V_1 should be approximately 90°. The numerical phase of V_1 is 54°, which compares closely to the theoretically derived phase value. This tendency is also shown in Figure 4 of Poole *et al.* (1988), in which the phase of V_1 varies from 0° to 90° while the height increases from 120 to 280 km. In contrast, for an X-mode wave the theoretical phases of V_{1L} and V_{1T} are -90° and 90°, respectively. Hence, the contributions of V_{1L} and V_{1T} cancel each other. Consequently, the theoretical phase of V_1 can take on any value and a comparison between the theoretical and numerical phase is unnecessary.

The numerical phase values of V_2 obtained for the O- and X-modes are -83° and -77°, while those for the theoretical values obtained here are -90° and -90°, respectively. This tendency is also apparent in Figure 4 of Poole *et al.* (1988), and Figures 5(b) and 5(c) of Sutcliffe and Poole (1989). This agreement suggests that the oscillations of Doppler velocity caused by the advection mechanism lag behind pulsations in the x component of the ground-level geomagnetic field by 90°.

The divergence mechanism has been further subdivided into compressional and gradient mechanisms by Poole *et al.* (1988). As shown in Table 1, the theoretical and numerical phase values of V_{3c} are different. This disagreement may arise because both b_{xI} and b_{zI} contribute oppositely to the Doppler velocity when the compression mechanism is considered. Nevertheless, b_{xI} is slightly larger than b_{zI} and therefore, the theoretical phase of V_{3c} should be ~90°. That the theoretical and numerical phases of V_{3g} are in close agreement shows that the oscillations of Doppler velocity caused by the gradient mechanism lag behind pulsations of the x component of the geomagnetic field by 90°. The numerical results of Poole *et al.* (1988) suggest that the amplitude of the Doppler velocity caused by the compression mechanism is much larger than that caused by the gradient mechanism. Consequently, the phase values of V_3 should be in close agreement with V_{3c}. Note that we have shown that the phase values of V_{3c} should be ~90° for both O- and X-mode waves and therefore, the theoretical phase values of V_3 should be ~90°. The tendency toward 90° of V_3 is shown by Figure 5(b) and 5(c) of Sutcliffe and Poole (1989), in which the numerical phase of V_3 varies from 0° to 90° while the height varies from 150 to 280 km. The data in these figures indicates that oscillations of Doppler velocity caused by the divergence (compression) mechanism lead pulsations of the northward (x) component of the geomagnetic field by 90°.

3.3 Theoretical Results and Observations

Observational evidence for TEC variations leading Doppler velocity oscillations by 90° can be found from the data presented by Davies and Hartmann (1976). Their Figures 3 and 4 show the variations of TEC and Doppler shift having periods of 50 seconds and 12 minutes during an event recorded at 2130 UT on October 7, 1974 at Boulder, Colorado. A 12-minute periodicity, with a minimum at ~2134 UT, can be observed in the measurement of Doppler velocity (obtained at a sounding frequency of 4.8 MHz), while a minimum occurs in the TEC data at ~2137 UT. The two records show that the variations in TEC lead Doppler velocity oscillations by ~3 minutes, which indicates that the TEC variations lead Doppler velocity oscillations by 90°.

Jarvis and Gough (1988), whose analysis is based on the theory of Poole and Sutcliffe (1987), indicated that the D component shows a tendency to be in anti-phase with Doppler velocity, consistent with a vertical motion of the ionosphere due to an $\mathbf{E} \times \mathbf{B}$ drift (advection mechanism). Furthermore, there was a tendency for the H and D components to be in phase with the Doppler velocity, which agrees with the concept of a direct compressional action of the hydromagnetic wave (divergence mechanism). As shown by the theoretical solutions derived here and evidence presented by Davies and Hartmann (1976), there is an intrinsic 90° phase difference between TEC variations and Doppler velocity oscillations. Consequently, the theory of Poole and Sutcliffe (1987) as applied by Jarvis and Gough should be re-examined.

Recently, Yumoto *et al.* (1989), who put forward the model developed by Poole *et al.* (1988), found the anti-phase relation between dH/dt pulsation with ~2-min period recorded at Onagawa (L=1.3) and the long-period V^* oscillation obtained at Kokubunji (ϕ=25.2°, Λ=205.8°) at 20:12UT on February 8, 1986 and concluded that an advection mechanism with -90° relation between V^* and H is observed.

On March 24, 1991, Doppler frequency shift oscillation obtained from a CW-HF Doppler sounding system and the H component of ULF pulsations recorded by a fluxgate magnetometer at Lunping (25°00'N; 121°10'E) showed phase differences of 15°-77° (Liu, *et al.*, 1993), which is consistent with the theoretical results due the compression mechanism.

4. SUMMARY

Beginning with work presented in the literature, we have developed a theoretical method to study the phase difference between TEC variations, Doppler velocity oscillations and ground-level geomagnetic pulsations. Under the assumption of sunspot maximum conditions, the theoretical results are compared with earlier numerical and observational results.. In summary, we have found:

(i) Variations in TEC caused by both the advection and compression mechanisms and pulsations of the northward component of the geomagnetic field are in phase and anti-phase;

(ii) The in-phase and anti-phase relationships between O- and X-mode Doppler velocity oscillations are caused by the magnetic mechanism, due to the transverse- and longitudinal-component of ionospheric geomagnetic pulsations;

(iii) Doppler velocity oscillations caused by the advection mechanism lag behind pulsations of the northward component of the ground-level geomagnetic field by 90°;

(iv) Doppler velocity oscillations caused by the compression mechanism lag behind and lead pulsations of the downward and northward components of the ground-level geomagnetic field by 90°;

(v) Variations in TEC lead Doppler velocity oscillations by 90°.

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REFERENCES

Davies, K., and G. K. Hartmann, 1976: Short-period fluctuations in total columnar electron content. *J. Geophys. Res.*, **81**, 3431-3434.

Dungey, J, W., 1963: Conference on the Ionosphere. Inst. Physics, London, Proc. Int., 230pp.

Hughes, W. J., 1974: The effect of the atmosphere and ionosphere on long period magnetospheric micropulsations. *Planet. Space Sci.*, **22**, 1157-1172.

Hughes, W. J., and D. J. Southwood, 1976: The screening of micropulsation signals by the atmosphere and ionosphere. *J. Geophys. Res.*, **81**, 3234-3240.

Jarvis, M. J., and H. Gough, 1988: Digital ionosonde observations of Pc3-4 pulsations across the plasmapause. *Planet. Space Sci.*, **36**, 733-745.

Klostermeyer, J., and J. Röttger, 1976: Simultaneous geomagnetic and ionospheric oscillations caused by hydromagnetic waves. *Planet. Space Sci.*, **24**, 1065-1071.

Liu, J. Y., Y. N. Huang, and F. T. Berkey, 1993: The phase relationship between ULF geomagnetic pulsations and HF Doppler frequency shift oscillations in 24 March 1991. *J. Geomag. Geoelectr.*, **45**, 109-114.

Menk, F. W., K. D. Cole, and J. C. Devlin, 1983: Associated geomagnetic and ionospheric variations. *Planet. Space Sci.*, **31**, 569-572.

Okuzawa, T., and K. Davies, 1981: Pulsations in total electron content. *J. Geophys. Res.*, **86**, 1355-1363.

Poole, A. W. V., and P. R. Sutcliffe, 1987: Mechanism for observed total electron content pulsations at mid latitudes. *J. Atmos. Terr. Phys.*, **49**, 231-236.

Poole, A. W. V., P. R. Sutcliffe, and A. D. M. Walker, 1988: The relationship between ULF geomagnetic pulsations and ionospheric Doppler oscillations: Derivation of a model. *J. Geophys. Res.*, **93**, 14656-14664.

Rishbeth, H., and O. K. Garriott, 1969: Introduction to Ionospheric Physics. Academic, San Diego, Calif.,U.S.A. 331pp.

Rishbeth, H., and W. B. Hanson, 1974: A comment on plasma "pile-up" in the F-region. *J. Atmos. Terr. Phys.*, **36**, 703-706.

Sutcliffe, P. R., and A. W. V. Poole, 1989: Ionospheric Doppler and electron velocities in the presence of ULF waves. *J. Geophys. Res.*, **94**, 13505-13514.

Tedd, B. L., K. D. Cole, and P. L. Dyson, 1989: The association between ionospheric and geomagnetic pulsations in the Pc3-4 range at mid-latitudes. *Planet. Space Sci.*, **37**, 1079-1094.

Watermann, J., 1983: Observations of correlated ULF fluctuations in the geomagnetic field and in the phase path of ionospheric HF sounding. *J. Geophys.*, **61**, 39-45.

Yumoto, K., K. Takahashi, T. Ogawa, and T. Watanabe, 1989: Sc- and Si-associated ULF and HF-Doppler oscillations during the great magnetic storm of February 1986. *J. Geomag. Geoelectr.*, **41**, 871-878.