ABSTRACT

This paper reviews studies on earthquake energies, seismic efficiency, radiation efficiency, scaled energy and energy-magnitude law conducted by Taiwan seismologists and foreigners who used seismic data from Taiwan to study these problems. Sufficient seismic and geodetic data permits energy measurements from the 20 September 1999 M7.6 Chi-Chi earthquake and its larger-sized aftershocks. The results provide significant information on earthquake physics. The issues in this review paper include measures of these physical quantities and related theoretical or analytical studies of these physical quantities made by both Taiwan’s seismologists and foreigners who used seismic data of Taiwan to study related problems.

Key words: Earthquake energies, Seismic efficiency, Radiation efficiency, Scaled energy, Energy-magnitude law


1. INTRODUCTION

After an earthquake ruptures the frictional stress, $\sigma(t)$, which is a function of time and slip on a fault plane, decreases from an initial $\sigma_o$ to a dynamic $\sigma_d$ and finally becomes $\sigma_f$ (see Fig. 1). In general $\sigma_d$ is equal to or smaller than $\sigma_f$ (Kanamori and Heaton 2000). $D_{max}$ is the maximum or total displacement. According to the slip- and rate-weakening frictional law, the frictional stress changes from $\sigma_o$ to $\sigma_d$ in a characteristic slip displacement, $D_c$ (Marone 1998; Wang 2002). The friction law that describes the frictional stress is complicated (cf. Ruina 1983; Marone 1998; Wang 2002). However, it can be approximated using a piece-wise linear function displayed in Fig. 1, which shows an example with $\sigma_d = \sigma_f$. The static stress drop $\Delta \sigma_s = \sigma_o - \sigma_f$ and the dynamic stress drop $\Delta \sigma_d = \sigma_o - \sigma_d$ are usually used to specify the change in stresses on a fault.

The strain energy, $\Delta E$, which results from tectonic loading, can release during an earthquake. The strain energy, $\Delta E$, per unit area can be approximated using the area of a trapezoid underneath the linearly decreasing stress versus slip function, i.e., the line segment AD in Fig. 1. The strain energy $\Delta E$ is transferred into, at least, three parts (see Fig. 1): the seismic radiation energy ($E_s$), fracture energy ($E_g$), and frictional energy ($E_f$), that is, $\Delta E = E_s + E_g + E_f$. $E_s$ is the energy radiated through seismic waves which leads to ground motions and can be detected by seismographs. $E_g$ is the energy used to extend the fault plane and cannot be measured directly from seismograms. $E_f$, which results from the dynamic friction stress, can generate heat. Because of incomplete data there are high uncertainties in measuring these energies, especially for $E_f$.

Two physical quantities, i.e., seismic efficiency and radiation efficiency, are defined directly from the four types of earthquake energies to represent source properties. The seismic efficiency, $\eta$, which is defined as the ratio of $E_s$ to $\Delta E$, i.e., $\eta = E_s/\Delta E$, has been long taken to present the level of seismic-wave radiation generated from an earthquake source. Kanamori and Heaton (2000) defined the radiation efficiency, $\eta_R$, as $\eta_R = E_s/(E_s + E_g)$. This parameter can be evaluated directly from seismograms. Venkataraman and Kanamori (2004) observed $\eta_R = 0.25 - 1$ for most earthquakes. Taking the seismic moment ($M_o$) into account, Kanamori (1977) defined the scaled energy as the ratio of seismic radiation energy to the seismic moment, i.e., $e_R = E_s/M_o$. It can be written as $(2\Delta \sigma_d - \Delta \sigma_s)/2$ (Kanamori and Heaton 2000). Gutenberg and Richter (1956) related the seismic radiation energy to earthquake magnitude:
log(E_s) = 11.8 + 1.5M_s (E_s in ergs). This equation is the so-called Gutenberg-Richter’s energy-magnitude law (abbreviated as the GR law hereafter), which is an important earthquake source scaling law.

Taiwan is located at the juncture of the Eurasian plate and the Philippine Sea plate (Tsai et al. 1977; Wu 1978; Lin 2002). The Philippine Sea plate has been moving north-westward at a speed of ~80 mm year$^{-1}$ (Yu et al. 1997) to collide with the Eurasian plate. The Okinawa Trough extends south-westward to approach Taiwan (Eguchi and Uyeda 1983). Active orogeny due to the collision of these two plates causes complex geological features and high seismicity in the region (from 119 - 123°E and 21 - 26°N). The complex tectonics in the region results in a non-uniform spatial earthquake distribution (Wang 1988, 1998; Wang and Shin 1998). High and heterogeneous seismicity in Taiwan makes the region serve as one of the best natural laboratories for seismological studies. Hence, seismicity studies have been conducted in Taiwan for more than one century (Wang 1998; Wang and Shin 1998). There are numerous types of seismic stations in the region (Wang 1989; Shin and Chang 2005). This makes studies of earthquake energies possible. On 20 September 1999 the M 7.6 Chi-Chi earthquake ruptured the Chelungpu fault, which is a ~100-km-long and east-dipping thrust fault, with a dip angle of 30°, in Central Taiwan (Ma et al. 1999; Shin and Teng 2001). The earthquake and its large aftershocks caused severe damage in Taiwan (Wang et al. 2005). A large number of seismological, geophysical and geological data were collected for earth scientists to study. From these data fruitful research results for the earthquake sequence and related problems have been made by earth scientists (Wang 2006b, 2010; Wang et al. 2005). The main shock energies and its larger-sized aftershocks were measured (Wang 2006a, 2010). The results provide significant information on earthquake physics.

This paper reviews the studies (including measures, methodologies, and theory) of earthquake energies, seismic efficiency, radiation efficiency, scaled energy, and energy-magnitude law made by Taiwan seismologists and foreigners who applied Taiwan seismic data to study earthquake energies. The main issues include the measures of these physical quantities for Taiwan earthquakes and the related theoretical or analytical studies of these physical quantities. To obtain completeness of this work, included also are the related studies for physical quantities in use done by the foreigners.

2. DESCRIPTION OF ENERGIES OF EARTHQUAKES

2.1 Strain Energy

The strain energy exerted by geotectonics and released during an earthquake can be written as (cf. Knopoff 1958):

$$\Delta E = \int u_{ij}(\sigma_{oij} + \sigma_{fij})v_{j}dS$$

\hspace{10cm} (1)

where $u_i$ is the slip along the i-th axis, $v_j$ is the unit vector normal to the fault plane and along the j-th axis, and $\sigma_{oij}$ and $\sigma_{fij}$ are, respectively, the initial (denoted by “o”) and final (shown by “f”) stress tensors, and S and dS are, respectively, the area and the unit area on the fault plane. Under some assumptions (Kostrov 1974; Dahlen 1977), an approximated formula of strain energy can be written as

$$\Delta E = (\sigma_0 + \sigma_f)\bar{\bar{u}}S/2$$

\hspace{10cm} (2)

where $\bar{\bar{u}}$ is the displacement on the fault plane and $(\ldots)_{ave}$ denotes the average quantities inside the parenthesis and S is the ruptured area. The values $\sigma_0$ and $\sigma_f$ cannot be determined just from seismological observations. When crustal deformation data are available, we can evaluate $\Delta E$.
Wang (2004) proposed a method to measure ΔE of an earthquake from the slip distribution of an earthquake source inversely from crustal deformation data. In his method the rotation components are excluded due to small values. He wrote ΔE as ΔE = ΔE_L + ΔE_W, where ΔE_L and ΔE_W denote the strained energies along the fault-striking (L) direction and the fault-dipping (W) one, respectively. From Eq. (2),

\[ \Delta E_L = \frac{(\sigma_{ul} + \sigma_{ul})}{2} u_L A \]

and \[ \Delta E_W = \frac{(\sigma_{uw} + \sigma_{uw})}{2} u_W S, \]

where \( u_L \) and \( u_W \) are the average displacements along the L- and W-directions, respectively. Define \( \sigma_L = (\sigma_{ul} + \sigma_{ul})/2 \) and \( \sigma_W = (\sigma_{uw} + \sigma_{uw})/2 \), thus leading to \( \Delta E_L = \sigma_L u_L S \) and \( \Delta E_W = \sigma_W u_W S \). Since \( \sigma_L \approx \mu(u_L/W) \) and \( \sigma_W \approx \mu(u_W/W) \), where \( \mu \) is the rigidity of the fault zone, \( \Delta E_L = \mu(u_L/L)u_L S, \Delta E_W = \mu(u_W/W)u_W S, \) and \( \Delta E = \mu(u_L/L)u_L + \mu(u_W/W)u_W S \). The errors due to approximation are \( L(d^2\sigma_L/dx^2) \) for \( \sigma_L \) and \( W(d^2\sigma_W/dy^2) \) for \( \sigma_W \). The values of \( d^2\sigma_L/dx^2 \) and \( d^2\sigma_W/dy^2 \) are unknown. However, it sounds reasonable to assume \( d^2\sigma_L/dx^2 \approx 0 \) and \( d^2\sigma_W/dy^2 \approx 0 \), when the variation in the stress field is low within the space domain in use. In practice, the stress field is considered to be constant inside a grid through the inversion procedure. This would lead to small errors. Except for the areas with abnormally large changes in displacements, the variation in slip on the fault is smooth, and thus, the higher-order derivatives of deformations would be small. In the practical inversion procedure the displacement on a grid is set to be a constant. This makes the higher-order derivatives of deformation be zero. Therefore, the difference between the estimated and real strain energies caused by excluding the two components should be small.

### 2.2 Seismic Radiation Energy

The seismic radiation energy, \( E_s \), is the energy radiated from the earthquake source through seismic waves. Rivera and Kanamori (2005) proposed a representation theory to describe the seismic radiation energy. In their theory, the strain energy and \( E_s \) are written as a function of the displacement and stresses in the fault zone. However, a simplified form is described below. In Fig. 1, \( E_s \) per unit area is the quantity inside triangle ACD. Assuming that during sliding the friction stress is almost constant acted on by dynamic friction, \( \sigma_0 \), with a dynamic stress drop \( \Delta \sigma \), the seismic energy is \( E_s = \Delta E - E_f - E_i = (\sigma_0 + \Delta \sigma)ds/2 - uS - 2GS \), where \( G \) is the fracture energy density as defined below. Making the additional assumption that the surface fracture energy is negligible, we obtain the simple expression

\[ E_s = M_o(2\Delta \sigma - \Delta \sigma)/2\mu \]  

(cf. Kanamori and Heaton 2000; Kanamori and Brodsky 2004), where \( M_o, \Delta \sigma, \) and \( \Delta \sigma \) are, respectively, the seismic moment, dynamic stress drop and static stress drop. From Eq. (3) it is clear that the seismic radiation energy contains only information concerning the stress change during the earthquake rupture, and no information concerning the total source area stress (cf. Scholz 1990). From the previous models \( E_s \) can be evaluated from the displacements and stresses on the fault plane (Boatwright 1980; Ide 2002; Favreau and Archuleta 2003).

On the other hand, \( E_s \) can be measured from seismic waves. Galitzin (1915) first measured the \( E_s \) value of the Pamir earthquake of 18 February 1911 from far-field seismic waves. Jeffrey (1923) corrected the formula used by Galitzin (1915) to measure \( E_s \). He calculated the total elastic wave energy in an earthquake spreading out spherically from a focus. Gutenberg and Richter (1942) suggested a simplified formula to calculate \( E_s \). They considered that at the epicenter the radiated energy arrives principally in a series of \( n \) equal sinusoidal waves of length \( \lambda \), amplitude \( A_n \), and period \( T_n \). The kinetic energy per unit volume is \( \rho(2\pi A_n/T_n)^{3/2}/4 \) where \( \rho \) is the density of the source area and the quantity in parentheses is the maximum velocity of a particle, and one factor \( 1/2 \) is due to averaging \( \sin^2(2\pi/T_n) \) over a period. If the wave velocity \( v \) is constant the mean energy in a spherical shell of volume \( 4\pi h^2 nh \) where \( h \) is the linear distance from the source. Hence, putting \( nT_n = t_0 \) and \( \lambda = vT_n \) leads to \( n\lambda = vt_0 \) and \( E_s = 4\pi h^2 vt_0 \rho (A_n/T_n)^2 = h^2 v^2 t_0 (a T_0)^2/4\pi \), where \( a \) is the acceleration, because of \( A_n = a T_0^2/4\pi \). In their calculations, they took \( v = 3.4 \text{ km s}^{-1} \) and \( \rho = 2.7 \text{ gm cm}^{-3} \). Note that the two ways used by Jeffereys (1923) and Gutenberg and Richter (1942) were too simplified to accurately measure the \( E_s \) value. Meanwhile, the corrections that are necessary to revise the measured value of \( E_s \), as mentioned below were not made in their studies.

Currently the seismic radiation energy is commonly measured from either recorded velocity seismograms or the velocity waveforms performed from the displacement seismograms or accelerograms based on the following expression:

\[ E_s = S_2 \rho B \int \hat{v}^2(t) dt = 2S_2 \rho B \int \hat{V}^2(f) df \]

(4)

where \( S_2 = 2\pi r^2 \) (\( r \) = hypocentral distance), \( \rho \) = density, \( B \) = S-wave velocity, \( v(t) \) = the velocity seismogram, and \( V(f) \) = Fourier Transform of \( v(t) \). \Es\ can be measured from three kinds of waves: the P-, S-waves, and Rayleigh waves (Boatwright and Fletcher 1984; Choy and Boatwright 1995; Pérez-Campos and Beroza 2001). However, several factors can influence the measures of \( E_s \). These factors include (1) instrumental response; (2) free surface amplification factor of 2; (3) seismic attenuation represented by the Q-value; (4) radiation pattern correction, i.e., \( R^{1/2} = (2/5)^{1/2} = 0.63 \); (5) directivity; (6) trapped-wave effect; (7) difference in the geological structures between the foot wall and hanging wall; (8) finite frequency bandwidth limitation effect (abbreviated the FFBL effect hereafter); and (9) site effect (especially for f > 3 Hz signals). Since the factors from (1) to (7) are less
complicated and well-known, only the FFBL effect and site effect are described below.

2.2.1 The Effect Due to Finite Frequency Bandwidth Limitation

The source spectra of earthquakes are mainly controlled by the low-frequency spectral level ($\Omega_s$) and corner frequency ($f_c$) (Haskell 1966; Aki 1967; Brune 1970). When $f > f_c$, the spectral amplitude decays in a power-law function like $f^{-\alpha}$. The scaling exponents are -2 and -3, respectively, referred to as the $\omega^2$ ($\omega = 2\pi f$) and $\omega^3$ source models. Huang and Wang (2002) observed that the scaling exponents of displacement spectra of the 1999 Chi-Chi, Taiwan, earthquake from the seismograms at nine near-fault stations increase from 1.63 - 3.04 from south to north. Hence, the two source models should be taken into account.

Let $d(t)$ and $v(t)$ be the source displacement and velocity, respectively. Their Fourier transforms are, respectively, $D(f)$ and $V(f)$. $D(f)$ can be approximated by $D_2(f) = \Omega_s[(1 + (ff_c)^2)$ for the $\omega^2$ model and $D_3(f) = \Omega_s[(1 + (ff_c)^3)$ for the $\omega^3$ model (cf. Beresnev and Atkinson 1997). Hence, the approximations of $V(f)$ are, respectively:

$$V_2(f) = 2\pi f\Omega_s[(1 + (ff_c)^2)$$

for the $\omega^2$ model; and

$$V_3(f) = 2\pi f\Omega_s[(1 + (ff_c)^3)^{\frac{3}{2}}$$

for the $\omega^3$ model. Figure 2 shows the log-log plots of the normalized, simplified velocity spectra, $V(f)$ versus $f$. Since $V_2(f) \sim f^3$ and $V_3(f) \sim f^4$ as $f \ll f_c$ and $V_2(f) \sim f^1$ and $V_3(f) \sim f^2$ as $f \gg f_c$, Eqs. (5) - (6) can be approximated individually by a piece-wise linear function as shown in Fig. 2. In the figure, the dashed and dotted lines, respectively, represent the $\omega^3$ and $\omega^2$ source velocity models.

In principle, the first integral in Eq. (4) is calculated from $-\infty$ to $+\infty$ in the time domain and the second one from 0 to $+\infty$ in the frequency domain. Define

$$I_v = \int v^2(t) \, dt = 2 \int V^2(f) \, df$$

This gives $I_v = 4\pi\rho\beta I_c$.

Ide and Beroza (2001) first pointed out the effect on measuring seismic radiation energy due to the FFBL, which is caused by windowing the source spectra in a frequency band from $f_1$ to $f_2$ as displayed in Fig. 2. The FFBL effect would change the source spectra in use, thus influencing the measured value of $E_v$.

Wang (2004) derived the formulas to present the FFBL effect based on the two models. In the followings, a subscript is taken ‘o’ to denote a quantity obtained through integration from $-\infty$ and $+\infty$ sec in the time domain or from 0 to $+\infty$ Hz in the frequency domain. Inserting Eqs. (5) and (6), respectively, into Eq. (7), with $f_1 = 0$ and $f_2 = \infty$, leads to $I_{V2o} = \Omega_s(2\pi f_c)^3/4$ for the $\omega^2$ model; and $I_{V3o} = \Omega_s(2\pi f_c)^3/16$ for the $\omega^3$ model. This gives $I_{V2o} = 4I_{V3o}$. Inserting Eqs. (5) and (6), respectively, into Eq. (7) and integrating from $f_1$ to $f_2$, lead to, respectively, $I_{V2} = I_{V2o}F_{V2v}$, and $I_{V3} = I_{V3o}F_{V3v}$, where $F_{V2} = (2\pi)^{-1}(-f_o/f_1)[1 + (f/f_1)] + (f/f_1)[1 + (f/f_1)]^2 + \tan^{-1}(f/f_c) - \tan^{-1}(f/f_2))$ for the $\omega^2$ model; $F_{V3} = (4\pi)^{-1}(-f_o/f_1)[1 + (f/f_1)]^2 + (f/f_1)[1 + (f/f_1)]^2 + \tan^{-1}(f/f_2) + \tan^{-1}(f/f_2)^{\frac{3}{2}} + (f/f_1)[1 + (f/f_1)]^2 - (f/f_1)^2/[1 + (f/f_1)]^2 - \tan^{-1}(f/f_2)/2)$ for the $\omega^3$ model. It is noted that when $f = 0$ and $f_\infty \rightarrow \infty$, $F_{V2} = 1$ and $F_{V3} = 1$, and, thus, $I_{V2} = I_{V2o}$ and $I_{V3} = I_{V3o}$.

Let $E_v$ and $E_c$ denote, respectively, the seismic radiation energy without and with, respectively, the FFBL effect. Hence, the energy ratio is $E_v/E_c = F_{V2v}$ for the $\omega^2$ model and $E_v/E_c = F_{V3v}$ for the $\omega^3$ model. Examples of the variations of $E_v/E_c$ with $f_1f_2 = 0.05 - 0.95$ and $f_1/f_2 = 2$ to 20, with a difference of 2, are shown, respectively, in Figs. 3 (for $E_v/E_c$) and Fig. 4 (for $E_v/E_c$), where the dotted line displays the energy ratio of 1 and also displayed are the maximum values for respective cases. Figures 3 and 4 express $E_v/E_c < 1$, with a maximum of 0.937, and $E_v/E_c < 1$, with a maximum of 0.999. Obviously, the FFBL effect yields under-estimates of seismic radiation energy. $E_v/E_c$ and $E_v/E_c$ both decrease with increasing $f_1/f_2$, and the amount of decreasing rate increases with $f_1/f_2$. The value of $f_1/f_2$ is in general higher for small earthquakes than for large ones. Hence, under-estimates of $E_c$ are higher for small earthquakes than for large ones.

Figure 3 shows that for fixed $f_1$, $E_v/E_c$ and $E_v/E_c$ decrease with increasing $f_2$ and increase with decreasing $f_1$. When $f_1/f_2 < 0.4$ for $E_v/E_c$ and $f_1/f_2 < 0.2$ for $E_v/E_c$, the curves are almost flat. This indicates that $f_1 = 0.4f_2$ for $E_v/E_c$ and $f_1 = 0.2f_2$ for $E_v/E_c$ are the respective optimum lower bounds for stable $E_c$. Figure 4 shows...
that $E_g/E_s$ and $E_d/E_{so}$ both increase with $f/f_c$. The curves are close to one another for $E_g/E_{so}$ when $f/f_c \geq 10$ and for $E_d/E_{so}$ when $f/f_c \geq 4$, thus indicating that $f_s = 10f_c$ for $E_g/E_{so}$ and $f_s = 4f_c$ for $E_d/E_{so}$ can lead stable $E_s$. For fixed $f_s$, increases in $E_g/E_{so}$ and $E_d/E_{so}$ with $f/f_c$ yield increases in the two ratios with $f_s$, thus indicating that an increase in $f_s$ improves estimates of $E_s$. The results obtained by Wang (2004) and Wang and Huang (2007) are consistent with those made by others (e.g., Boore 1986; Di Bona and Rovelli 1988; Singh and Ordaz 1994; Ide and Beroza 2001).

Figures 3 and 4 show that for fixed $f_c$, the energy ratios decrease with increasing $f/f_c$ and thus they increase with $f_c$. This implies that the FFBL effect in the low-frequency regime gives a greater underestimate of $E_s$ for events with lower $f_c$ than for those with higher $f_c$. This effect is stronger for the $\omega^{-3}$ model than the $\omega^{-2}$ model.

### 2.2.2 The Site Effect

Observations show that seismic waves are amplified at sedimentary sites (Wang et al. 2002; Huang et al. 2005, 2007, 2009), because the seismic waves are amplified when they propagate through the low-shear-velocity and low-density layers. The amplification of seismic waves is usually a function of frequency and stronger at a soil site than at a rock one. The site classification criteria used in the USA (see Huang et al. 2005, 2007, 2009) are based on $V_{so}$, which is the averaged shear velocity from the ground surface to 30-m depth: the Class-A site with $V_{so} > 1500$ m sec$^{-1}$, the Class-B one with $V_{so} = 760 - 1500$ m s$^{-1}$, the Class-C one with $V_{so} = 360 - 760$ m s$^{-1}$, the Class-D one with $V_{so} = 180 - 360$ m s$^{-1}$, and the Class-E one with $V_{so} < 180$ m s$^{-1}$. Based on these criteria, numerous strong-motion stations are built on the soil sites. The site amplifications could result in over-estimates of $E_s$. Hence, it is necessary to correct the site effect. From the quarter-wavelength approximation method proposed by Boore and Joyner (1997) and Huang et al. (2005, 2007) evaluated the frequency-dependent site amplifications at 87 free-field strong-motion stations in central Taiwan from the velocity and density structures constructed from well-logging data measured in shallow holes near station sites and the average velocity models for the area inferred from earthquake data by Chen et al. (2001) and Sato et al. (2001). Well-logging velocities measured at shallow and deep holes suggest velocity reliability, at least in the upper 2000 m, inferred by earthquake data. Huang et al. (2009) evaluated the frequency-dependent site amplifications from well-logged data in the Taipei Basin. Their results show three key points: (1) there is no Class-A site and only a few Class-B sites in the study area; (2) the site amplifications are the largest at Class-E sites, intermediate at Class-D, and smallest at Class-C; and (3) in spite of wave attenuation, the site amplification increases with frequency for all classes. Point (1) suggests that site amplification removal is strongly necessary for measuring $E_s$ especially from strong-motion seismograms. Together with regional geology, point (2) leads to site amplification being larger in the Western Plain with thick Holocene alluvium than in the Western Foothill with Pleistocene and Miocene formations.

### 2.3 Fracture Energy

The fracture energy $E_g$ is the energy used to extend the fault plane and can be influenced by numerous factors. Husseini et al. (1975) related $E_g$ to the stress drop and characteristic radius of a fault. From laboratory experiments and numerical simulations, Fialko and Rubin (1997) observed an increase in fracture energy with confining pressure. However, it is difficult to examine this correlation just from seismological observations. Kanamori and Heaton (2000) considered that $E_g$ can be evaluated using the following equation:

$$E_g = [(1 - v_s/\beta)/(1 + v_s/\beta)]^{1/2} \Delta \sigma S \Delta \sigma / 2$$

where $v_s$ and $\beta$ are, respectively, the rupture and S-wave
velocities. This equation is valid only for a crack-like rupture model (Tinti et al. 2005), and $E_c$ computed from Eq. (8) is an average global value, because $\Delta \sigma_3$ and $\bar{u}$ are both average values over the fault plane. $E_c$ obviously depends on $v_d/\beta$, and is much smaller than $\Delta E$ because of $v_d/\beta = 0.75 - 0.85$ (Kanamori and Heaton 2000). $G = E_c/S$ is defined as the fracture energy density (per unit area). From the definition, $G$ must be a local parameter. However, in practice only the $G$ value on a certain portion of a fault plane or the whole fault plane can be measured and thus only the global $G$ average is calculated. In general, $G$ is $10^6$ - $10^7$ J m$^{-2}$ for earthquakes (see Scholz 1990; Ide 2003; Rice et al. 2005; Tinti et al. 2005).

2.4 Frictional Energy and Heat

From $E_c = \Delta E - (E_t + E_r)$, we can obtain the frictional energy. On a fault area of $S$, heat produced by $\sigma_3$ in an average displacement $\bar{u}$ during faulting is $E_t = \sigma_3 \bar{u} S$, and $E_r$ yields a temperature rise of $\Delta T$. Assuming that heat is distributed within a layer of thickness $h$ around the ruptured plane, $\Delta T$ is

$$\Delta T = E_t/C \rho Sh \tag{9}$$

where $C$ and $\rho$ are, respectively, the specific heat and density (Kanamori and Heaton 2000). For crustal rocks, $C = 10^3$ J/kg-°C and $\rho = 2.6 \times 10^3$ kg m$^{-3}$. The heat strength is defined as $Q = E_t/C \rho S = \Delta T h$.

In order to study the relationship among frictional strength, pore pressure and heat, Wang (2006b) constructed a 2-D (thrust) faulting model with frictional heat. A brief description about his model is given below. The lithostatic pressure $\sigma_{ls}$ at the average fault depth, $H$, is $\rho g H$. The (maximum) horizontal principal stress $\sigma_1$ is $\rho g H$ plus an additional tectonic stress, and the (minimum) vertical principal stress is $\sigma_2 = \sigma_{ls}$. The normal and shear stresses, i.e., $\sigma_1$ and $\sigma_3$, on the fault plane with a dip angle of $\theta$ are both a function of $\sigma_1$ and $\sigma_2$. The relation of $\sigma_3$ versus $\sigma_1$ is in the form: $\sigma_3 = \mu_3(\sigma_1 - p_u)$, where $\mu_3$ is the frictional coefficient and $p_u$ is the pore pressure. Let $p_n = \gamma \rho g H$, where $\gamma$ is the pore-fluid factor (cf. Sibson 1992). At shallow depths, where the fluid gradient is hydrostatic and $\gamma$ is the ratio of fluid to rock density, typically -0.4. At depths, where the fluid pressure may become suprahydrostatic, $\gamma > 0.4$, with an extreme of $\gamma \rightarrow 1$. When a fault zone breaks, $\sigma_3 = (\sigma_1 - \mathcal{I})$ drops to $\sigma_3$. Since $\mathcal{I} = \mu_3(1 - \gamma) \sigma_3$, $\mathcal{I} = \mu_3(1 - \gamma)$ behaves like the effective frictional coefficient. Based on Anderson theory of faulting (cf. Turcotte and Schubert 1982), Wang (2006b) related $\Delta T$ to several parameters on the fault in the following form:

$$\Delta T = Q/h = \Xi \mu_3(1 - \gamma) g H \sin(2\theta)/hC[(1 + \mu_3)^{1/2} - \mu_3] \tag{10}$$

from this equation we can evaluate the pore pressure on the fault plane.

2.5 Seismic Efficiency

The seismic efficiency, $\eta$, is defined as the ratio of $E_r$ to $\Delta E$, i.e., $\eta = E_r/\Delta E$, has been long taken to present the level of seismic-wave radiation generated from an earthquake source. The seismic radiation energy can be approximated by $\eta = \Delta \sigma_3/(\sigma_1 + \sigma_3)$ (cf. Scholz 1990). Although the dynamic stress drop, $\Delta \sigma_3$, can be determined from seismograms, the total stress must be evaluated from non-seismic data. Hence, the seismic radiation energy, which obviously depends upon the total stress, cannot be determined only from seismological observations. When $\Delta \sigma_3$ is constant, $\eta$ decreases with increasing total stress. Hence, the seismic efficiency can reflect the regional tectonics. Savage and Wood (1971) assumed $\eta \leq 0.07$. Spottswoode and McGarr (1975) reported $\eta \leq 0.01$ for the mine tremors. Boatwright (1978) reported $\eta = 0.08$ for an $M_L$ 1.5 event. Kikuchi (1992) reported $\eta = 0.012$ - 0.22 for 27 large earthquakes and stated that deep events have a smaller value than shallow ones. Kanamori et al. (1998) gave $\eta = 0.04$ for the 1994 deep Bolivia earthquake. From laboratory experiments and mining-induced events ($M = 1.9$ to $3.3$), McGarr (1994, 1999) hypothesized $\eta \leq 0.06$ and stated that this hypothesis holds for both small and large events. However, those authors estimated $\eta$ mainly from seismic data under some assumptions. I assume that the slip distribution inferred from seismic data cannot completely reflect the static strain field on the fault plane, which is the basis for measuring $\Delta E$. The seismograms used for source inversion usually consist only of shorter-period signals. Hence, their previous estimates of $\eta$ are problematic.

2.6 Radiation Efficiency

As mentioned above, the uncertainty of evaluating $\eta$ is high due to the difficulty in accurately measuring $\Delta E$. Hence, Kanamori and Heaton (2000) defined a new parameter, i.e., the radiation efficiency, $\eta_R$, which is $\eta_R = E_r/(E_t + E_r)$. This parameter can be evaluated directly from seismograms. Venkataraman and Kanamori (2004) observed $\eta_R = 0.25$ - 1 for most of earthquakes.

Kanamori (2004) related the radiation efficiency to the grain size and physical properties of slip zone on a fault using the following approximated formula:

$$\eta_R = 1/[1 + 6\lambda G_s(t_s/D_{max})/\mu_e d] \tag{11}$$

where $\lambda$ is the correction for grain roughness, $G_s$ is specific fracture energy, $t_s/D_{max}$ is the ratio of the slip thickness ($t_s$) to the total displacement ($D_{max}$), $\mu_e$ is the scaled energy which was defined by Kanamori (1977) and will be explained below, and $d$ is the average grain size. Obviously, $\eta_R$ is
The radiation efficiency is strongly affected by the variation in shear stress with slip (see Fig. 1). Wang (2009) considered thermal pressurization to be a significant mechanism in controlling the variation in shear stress, thus influencing \( \eta_r \). He derived the formula for \( \eta_r \) as a function of slip, \( \delta \), based on the two end-member models of thermal pressurization, i.e., the adiabatic-undrained-deformation (AUD) and slip-on-a-plane (SOP) models, proposed by Rice (2006). His formula are

\[
\eta_{\text{AUD}} = 1 - \frac{2[1 - (1 + \delta / \delta_c) \exp(-\delta / \delta_c)]}{[1 - \exp(-\delta / \delta_c)]^2} \tag{12}
\]

for the AUD model and

\[
\eta_{\text{SOP}} = 1 - \frac{2[1 - (\delta / \delta_c) \exp(-\delta / \delta_c)]^2}{[1 - \exp(-\delta / \delta_c)]^2} \tag{13}
\]

for the SOP model. Obviously, the radiation efficiency is mainly controlled by \( \delta / \delta_c \) for the AUD model and by \( \delta / \delta_c \) for the SOP model. Equations (12) and (13) show that \( \eta_{\text{AUD}} \) and \( \eta_{\text{SOP}} \) are zero when \( \delta = 0 \) and 1 when \( \delta \) approaches infinity.

The controlling parameters of the AUD and SOP models are, respectively, \( \delta_c \) and \( \delta_c \) which are individually in terms of thermal, mechanical, and hydraulic parameters of fault rocks. Modeled results suggest that thermal pressurization controls the variation in shear stress with slip and thus influence the radiation efficiency. Results show that \( \eta_r \) increases with \( \delta \). The increasing rate of \( \eta_r \) with \( \delta \) is high at small \( \delta \) and low at large \( \delta \). This indicates that \( \eta_r \) varies very much with \( \delta \) for small earthquakes and only slightly depends on \( \delta \) for large events. For the two end-member models, \( \eta_r \) increases with decreasing \( \delta_c \) (or \( \delta_c \)). When \( \delta_c = \delta_c \), \( \eta_r \) is higher for the AUD model than for the SOP model.

3. MEASURES OF EARTHQUAKE ENERGIES IN TAIWAN

3.1 Early Studies

Based on the GR law, Hsu (1971) first measured the temporal variations in \( E_s \) of \( M \geq 5 \) earthquakes in the Taiwan region from 1936 - 1969. The total release of \( E_s \) during the study period of time for the region was \( 1.062 \times 10^{24} \) ergs for \( M \geq 5 \) earthquakes and \( 0.956 \times 10^{24} \) ergs for \( M \geq 6 \) events. The mean annual energy release rate during the study period of time for the region was \( 1.062 \times 10^{18} \) ergs year\(^{-1}\) for \( M \geq 5 \) earthquakes and \( 0.956 \times 10^{18} \) ergs year\(^{-1}\) for \( M \geq 6 \) events. The latter is about one hundreth as much as the rate (\( \approx 2.85 \times 10^{24} \) ergs year\(^{-1}\)) for the global earthquakes. Hsu (1973) re-estimated the mean rate of \( E_s \) of \( M \geq 6 \) earthquakes, and his new value is \( 4.174 \times 10^{22} \) ergs year\(^{-1}\), which is about 1.6 times of the previous value. However, the earthquake magnitude scale used by Hsu (1971, 1973) was Hsu’ magnitude, \( M_H \), rather than the surface-wave magnitude, which appears in the GR law. Wang (1992) showed that \( M_H \) relates to \( M_s \) in the form of \( M_s = -0.95 + 1.15M_H \), and thus \( M_H > M_s \) when \( M_s < 6.3 \) and \( M_H < M_s \) when \( M_s > 6.3 \). This indicates that \( M_s \) was over-estimated for \( M_H < 6.3 \) earthquakes and under-estimated for \( M_H > 6.3 \) events by Hsu (1971, 1973). Chen and Wang (1985) measured the \( E_s \) of \( M \geq 4 \) earthquakes occurred during the 1973 - 1984 period in the Taiwan region with numerous units of \( 20^\circ \times 20^\circ \). Since they used the duration magnitude for the GR law, the calculated value of \( E_s \) should be revised.

3.2 1999 M, 7.6 Chi-Chi Earthquake

On 20 September 1999, the M, 7.6 Chi-Chi earthquake ruptured the Chelungpu fault, which is a ~100-km-long and east-dipping thrust fault, with a dip angle of 30°, in central Taiwan (Ma et al. 1999; Shin and Teng 2001). The epicenter, fault trace, and the fault plane are displayed in Fig. 5. In 2000, two shallow boreholes near the Chelungpu fault (see Figs. 5 and 6) were drilled (cf. Tanaka et al. 2002; Hung et al. 2007). The distances from the drilling site to the fault trace are 500 and 250 m, respectively, for the northern and southern boreholes. From core samples, two fractures zones can be recognized. Hung et al. (2007) stated that the two boreholes encountered the fault plane of the event, and assumed that the possible fracture zone of the Chi-Chi earthquake is at 225 - 330 and 177 - 180 m, respectively, in the northern and southern boreholes. The main results were reported by several authors (Otsuki et al. 2001; Tanaka et al. 2002; Hung et al. 2007; Wang 2010).

Kano et al. (2006) measured he temperature rise in the two shallow boreholes (see Fig. 6) about 1.4 years after the earthquake. The peak temperature values on the fault plane are 0.5 and 0.1°C, respectively, in the southern and northern
boreholes. The temperature rise decreases with increasing distance from the fault plane as described by a 1-D cooling equation, from which Mori (2004) estimated the frictional coefficients. Results are: (1) 0.7 - 1.0, with an average 0.85, at the 182-m depth in the south and 0.1 - 0.2, with an average 0.15, at the 320-m depth in the north; and (2) an average 0.45 for the two segments.

In 2005 the Taiwan Chelungpu-fault Drilling Project (TCDP) was launched (Song et al. 2007a), and thus two deep holes, i.e., Hole-A and Hole-B, with depths of ~2000 m were drilled cutting across the fault plane (see Fig. 6). The two holes are located 40 m apart. Both are located inside the solid circle of Fig. 5. The fault zone, denoted by the FZA1111, is located at the depth of ~1111 m below the ground surface. Kano et al. (2006) measured temperatures, with a resolution of 0.003°C, inside Hole-A in September 2005, six years after the earthquake, i.e., \( t = 1.9 \times 10^8 \) sec. They plotted a spatial distribution of temperature rise, \( \Delta T \), between -40 and +40 m, i.e., the line segment denoted by TT' in Fig. 6, with the center on the fault plane on which the maximum value is 0.06°C.

![Fig. 5. A figure to show the epicenter (in a solid star), the surface trace of the Chelungpu fault (in a solid line), the fault plane (bounded by four dashed lines), the nine near-fault seismic station sites (in open triangles), and the borehole sites (in solid circles). The northern and southern segments of the fault are separated at a locality near station TCU065 (after Wang 2006b).](image)

![Fig. 6. Structural profile across Hole A [reproduced from Hung et al. (2007)]. Line segment TT' displays the depth range within which temperature was measured by Kano et al. (2006) (after Wang 2006b).](image)
3.3 Strain Energy

As mentioned above, Wang (2004) proposed a method to measure ΔE of the earthquake from the slip distribution inferred by Domínguez et al. (2003). For the whole Chelungpu fault, Wang (2004) obtained ΔE = 3.206 × 10^{24} ergs, which is equivalent to M = 8.5 based on the GR law. On the two segments of the Chelungpu fault, the strain energies are ΔE_{N} = 2.341 × 10^{24} ergs and ΔE_{S} = 0.865 × 10^{24} ergs for the northern and southern segments, respectively. The related surface-wave magnitudes are M_{w} = 8.4 and 8.1 for the northern and southern segments, respectively.

3.4 Seismic Radiation Energy

Ma et al. (2000, 2001) estimated three related source parameters, i.e., M_{0} = 2.2 × 10^{27} dyne-cm, Δσ/Δt = 2.0 × 10^{7} dyne cm^{-2}, and Δσ = 1.1 × 10^{7} dyne cm^{-2}, from seismic data. This gives E_{S} = 8.39 × 10^{23} ergs based on Eq. (3). From teleseismic data, Venkataraman and Kanamori (2004) obtained E_{S} = 8.8 × 10^{23} J and η_{R} = 0.8 under some assumptions. From the GR law, Wang (2006b) obtained E_{S} = 6.31 × 10^{22} ergs (or 6.31 × 10^{15} J). Obviously, the values of E_{S} evaluated from seismic data are higher than that directly calculated from the GR law. This might be due to the reasons that Ma et al. (2000, 2001) and Venkataraman and Kanamori (2004) did not remove the FFBL and site effects from recorded seismograms.

Hwang et al. (2001) measured E_{S} from the seismograms recorded at nine near-fault seismic stations. Wang (2004) revised their values by eliminating the FFBL effect. For the whole Chelungpu fault, he obtained E_{S} = 4.307 × 10^{23} ergs, which is higher than that calculated from the GR law and equivalent to M_{t} = 8.0 and energy of ~676 atomic bombs. Obviously, the M_{t} value calculated from E_{S} is higher than M_{w} = 7.6 that was measured from the maximum ground vertical amplitude by the United States Geological Survey (USGS). For the two segments of the fault, E_{S_{N}} = 3.981 × 10^{23} ergs for the northern segment and E_{S_{S}} = 0.326 × 10^{23} ergs for the southern segments. The related values of M_{t} are 7.9 and 7.1, respectively. The E_{S} values are equivalent to ~622 and ~54 atomic bombs, respectively.

From the values of E_{S}, ΔE as mentioned above, Wang (2004) obtained the seismic efficiency of the 1999 Chi-Chi earthquake: η = 0.137 (or 13.7%) for the whole fault, η_{N} = 0.169 (or 16.9%) for the northern segment, and η_{S} = 0.038 (or 3.8%) for the southern segment. The η_{N} and η_{S} values lead to about 80% of ΔE_{N} and 97% of ΔE_{S} were transferred into the non-seismic radiation energies, mainly including E_{S} and E_{T}. In addition, results cannot completely fit the McCarr’s η ≤ 0.06 hypothesis.

Hwang (2012) measured the radiated seismic energy of the Mt 6.4 JiaSian earthquake of 4 March 2010 from teleseismic waves. His measured values are E_{S} = 2.91 × 10^{23} J and M_{t} = 2.17 × 10^{18} Nm, which is associated with M_{w} = 6.15. Obviously, the estimated seismic-moment magnitude is lower than the local magnitude.

3.5 Fracture Energy

From teleseismic data, Venkataraman and Kanamori (2004) obtained E_{r} = 0.88 × 10^{16} J and η_{R} = 0.8, thus leading to E_{S} = 0.22 × 10^{16} J for the overall fault plane. To calculate E_{S} and G of the Chi-Chi earthquake, Wang (2006b) took the (v/β)_{R}, Δσ/Δt values and S from Huang et al. (2001), Ma et al. (2001), and Wang (2004). The related parameter values are: (v/β)_{R} = 0.75, Δσ_{SS} = 6.52 MPa, D_{S} = 4.88 m, and S_{S} = 4.551 × 10^{8} m² for the northern segment; and (v/β)_{R} = 0.80, Δσ_{IN} = 29.7 MPa, D_{S} = 7.15 m, and S_{S} = 3.615 × 10^{8} m² for the southern segment. The E_{S} estimate depends on the D_{S} value. Wang (2006b) assumed that D_{S} = 1 m is acceptable for the southern segment and D_{S} should be in between 1.8 - 3.7 m for the northern segment. Hence, the E_{S}, G, and η_{R} values are: E_{S} = 0.15 × 10^{16} J, G_{S} = 0.33 × 10^{12} J m², and η_{RS} = 0.69 for the southern segment and 0.95 × 10^{16} J < E_{S} < 1.99 × 10^{16} J, 2.59 × 10^{7} m² < G_{S} < 5.34 × 10^{8} m², and 0.67 < η_{RS} < 0.81. The related parameter values are higher on the northern segment than on the southern segment.

From local seismograms, Zhang et al. (2003) evaluated the G values. Their results show that G increases from south to north, and G_{S} = 10^{-8} - 10^{-7} J m² in the south and G_{S} up to 3 × 10^{8} J m² in the north. Their values are about one-order-of-magnitude larger than those of Wang (2006b). From the core sample on the 1111-m slip zone of a 2000-m deep hole, Ma et al. (2006) observed that the thickness of the slip zone is about 0.02 m and the grain size is in the range (50 - 1000) × 10^{-9} m. From the grain size, they obtained average G = 4.8 × 10^{8} J m², which is about one fifth of that from Wang (2006b) and one-order-of-magnitude smaller than those estimated by Zhang et al. (2003). Obviously, the G_{S} values from Wang (2006b) seem better than those from the other two groups.

3.6 Frictional Energy and Heat

Mori (2004) inferred frictional energy for the whole fault from model computations. His value is E_{f} = 3.6 × 10^{16} J. The values of Q_{f} and Q_{c}, which are, respectively, the heat strength at the southern and northern shallow boreholes, and (ΔT) = (102/h)°C and (ΔT) = (154/h)°C. As mentioned above, Venkataraman and Kanamori (2004) obtained E_{c} = 0.88 × 10^{16} J and E_{c} = 0.22 × 10^{16} J for the overall fault plane, thus leading to E_{S} + E_{T} = 1.10 × 10^{16} J, which is one fifth of E_{S} + E_{T} = 5.47 × 10^{16} J from Wang (2006b). Using the ΔE value from Wang (2006b) and the E_{S} + E_{T} = 5.47 × 10^{16} J value from Venkataraman and Kanamori (2004), Wang (2006b) obtained E_{f} = 3.09 × 10^{17} J, which is 8.57 times higher than 0.36 × 10^{17} J from Mori (2004), and 1.17 times higher than 2.65 × 10^{17} J from Wang (2006b).
related Q values are 144.3°C-m from Venkataraman and Kanamori (2004) and 123.9°C-m from Wang (2006b), with a difference of 20.4°C-m. The difference is clearly small.

Based on the 1-D heat conduction equation described below, they used several modeled spatial distributions of ΔT to fit observed data and then evaluated the optimum values of heat strength \( Q = 1.5°C\times m \) and thermal diffusivity \( \alpha = 3.4 \times 10^{-7} \text{ m}^2\text{s}^{-1} \). Since their optimum model of ΔT fits the observations in a large spatial range -40 to 40 m, the inferred value of \( \alpha \) must be the average of wall rocks, because the thickness of primary slip zone (PSZ) identified by Ma et al. (2006) is only 0.12 m. Tanaka et al. (2007) measured \( \alpha \) directly from the core samples of PSZ. Their results are (0.8 - 2.0) \times 10^{-7} \text{ m}^2\text{s}^{-1}, with 1.0 \times 10^{-6} \text{ m}^2\text{s}^{-1} in the major slip zone (MSZ) identified by Ma et al. (2006). Obviously, their values are about 3 times larger than that inferred by Kano et al. (2006). This made heat diffusion faster in the fault zone than in wall rocks.

Kano et al. (2006) measured the temperature in the depth range -40 to 40 m with respect to the fault zone at the FZA1111 site six years after the earthquake. With ΔT = 0.06°C at \( x = 0 \), they inferred \( Q = 1.5°C\times m \) and \( \alpha = 3.4 \times 10^{-7} \text{ m}^2\text{s}^{-1} \). On the other hand Tanaka et al. (2007) assumed that the variation in thermal conductivity between the fault-zone materials and wall rocks caused the spatial variations in temperature measured by Kano et al. (2006). Hence, the Q value inferred by Kano et al. (2006) is questionable.

Tanaka et al. (2006, 2007) measured the thermal properties of the fault zone materials across the Chelungpu fault zone activated by the 1999 Chi-Chi earthquake using the drilled core penetrating the fault zone at around 1100 m depth. The fault zone contains four distinct fracture zones, each of which includes thin slip zones. Thermal conductivity lies between 1.0 - 3.0 W m-1 K-1 and shows the lowest value at the slip zones. Thermal diffusivity (\( \alpha \)) varies between 0.8 \times 10^{-8} and 2.0 \times 10^{-6} m^2 s^{-1}, and is relatively low at the slip zones. Density (\( \rho \)) varies between 2200 - 2800 kg m^{-3} and shows the lowest values at a particular slip zone (1110 m depth). Specific heat (c) is calculated using the above data resulting in values from 300 - 1000 J kg^{-1} K^{-1}, and lowest values for slip zones. Using these data and spectral gamma ray logs, reported positive thermal anomalies at the slip zones are re-examined whether they are regarded as residual heat from friction by faulting.

From laboratory experiments Hirono et al. (2007, 2008) interpreted \( \sigma_l = 1.37 \text{ MPa} \) for the black gouge at the FZB1136 of Hole-B. Since the FZB1136 is equivalent to the FZA1111 of Hole-A and almost the same black gouges exist in the fault zones of the two holes, their value is used here. From \( \sigma_l = 0.8 \sigma\), we have \( \sigma = 1.1 \text{ MPa} \). Since the study site is close to the ground surface, D almost equals \( D_p = 4.24 \text{ m} \). Tanaka et al. (2007) obtained \( C_v = 300 \text{ J kg}^{-1} \text{ °C}^{-1} \) and \( \rho = 2200 \text{ kg m}^{-3} \) for the MSZ. Inserting into Eq. (3) these values leads to \( Q = 7.0°C\times m \), which is about 4.7 times higher than that inferred by Kano et al. (2006). Using the previous data, Wang (2011) calculated the heat strength on the Chelungpu fault plane at a depth of 1111 m from relevant data obtained from Hole-A. The calculated value is 7.0°C-m, and is larger than that inferred by Kano et al. (2006), whose evaluation was based on a smaller value of thermal diffusivity of the wall rocks. The thermal history modeled from the 1-D heat conduction model, with the values of thermal diffusivity evaluated in a temperature range based on the Debye law, assumes that frictional heating occurred mainly in a very thin layer, < 5 mm, which is inside the black materials found around the fault plane. This heated layer had a larger thermal diffusivity than wall rocks and was the least deformed part of the fault zone. Calculated results exhibit that the temperature \( T \) increases from the ambient value of \( T_a = 46.5°C \) at \( t = 0 \) to a peak value \( T_{peak} \) of ~1100°C at the rise time \( t \), of ~2.5 sec and then decreases with increasing \( t \). Obviously, the frictional heat dissipated rapidly during the earthquake. This provides an answer to the so-called heat flux paradox (Lachenbruch and Sass 1980). There is no high thermal anomaly during faulting due to a remarkable decrease in the effective frictional stress.

Chemical analyses of pseudotachylites and clay minerals (including smectite, illite, kaolinite, and chlorite) of core samples and temperatures measured about six years after the earthquake in a 2000-m hole, which crosses the fault plane, Wang (2011) proposed a positive correlation between the spatial distribution of clay minerals and temperature rise caused by frictional heating during the earthquake. Pseudotachylites could be formed in the heated layer in a very short time interval, < 0.3 sec, immediately after faulting. Hence, the amount of pseudotachylite is tiny as observed by Song et al. (2007b). The amount of pseudotachylite and smectite, which was devitrified from black material glasses, is ~85% of the clay minerals inside and low outside the MSZ proposed by Ma et al. (2006). The clay minerals outside the MSZ, < 0.02 m, were very stable during faulting, because of \( T < 150°C \).

Based on the 2-D faulting model proposed by Wang (2006b), Wang (2011) evaluated the pore fluid pressure on the depth of 1111 m at Hole-A during faulting from the values of temperature rise and thermal and mechanical parameters at the hole. A difference of 10° in the dip angle only yields a small difference in the interpreted pore fluid pressure. The estimated value of the pore-fluid factor is 0.94, thus leading to a pore fluid pressure of 22.5 MPa. Results suggest that the fault zone could have been suprahydrostatic during faulting. The suprahydrostatic pressure reduced the effective friction coefficient and thus decreased frictional heating on the fault plane. This study provides possible causes of a low heat flow on a fault plane.

Wang (2011) assumed that quartz plasticity could be formed in the MSZ when \( T > 300°C \) after the study site ruptured. Quartz plasticity could lubricate the fault plane at higher \( T \) and yield viscous stresses to resist slip at lower \( T \).
The shear zone with quartz plasticity would be localized in a 5-mm thick heated layer.

### 3.7 Radiation Efficiency

From local seismograms, Wang (2006b) obtained the optimum values: (1) \( E_0 = 0.15 \times 10^{15} \text{ J} \), \( \eta_0 = 0.69 \), and \( G = 0.33 \times 10^3 \text{ J} \cdot \text{m}^{-2} \) for the southern segment; and (2) \( E_0 = 1.99 \times 10^{15} \text{ J} \), \( \eta_0 = 0.67 \), and \( G = 5.34 \times 10^3 \text{ J} \cdot \text{m}^{-2} \) for the northern segment. From teleseismic data, Venkataraman and Kanamori (2004) obtained \( E_0 = 0.88 \times 10^{16} \text{ J} \) and \( \eta_0 = 0.8 \). Ma et al. (2006) applied Eq. (11) to estimate the radiation efficiency at the drilled site. The values of \( G_c, T/D \), and \( d \) measured from the TCDP by Ma et al. (2006) are: \( G_c = 1 \text{ J} \cdot \text{m}^{-2} \), \( (T/D) = 4 \times 10^4 \) (due to \( T = 12 \text{ cm} \) and \( D = 300 \text{ m} \)), and \( d = 1.86 \times 10^7 \text{ m} \). The common value of \( \mu \) for crustal rocks is \( 3 \times 10^4 \text{ Pa} \). The value of \( \lambda \) ranges in general from 5 - 22 (cf. Wilson et al. 2005). Ma et al. (2006) selected \( \lambda = 6.6 \) for calculations. Consequently, the value of \( \eta_0 \) estimated by them is 0.88, which is close to \( \eta_0 = 0.8 \) for the whole fault plane of the Chi-Chi earthquake estimated by Venkataraman and Kanamori (2004) from teleseismic data and larger than \( \eta_0 = 0.67 \) for the northern fault plane evaluated by Wang (2006b) from local seismograms. For comparison, Wang (2006b) also used Eq. (11) with the upper bound of \( \lambda \), i.e., 22, to calculate \( \eta_0 \). He obtained \( \eta_0 \) = 0.68 which is close to Wang’s \( \eta_0 = 0.67 \). Ma et al. (2006) inferred the maximum displacement, \( \Delta_{\text{max}} \), at the study to be 8.3 m. Wang (2006b) pointed out that when \( \Delta_{\text{max}} < 10.7 \text{ m} \), thermal pressurization plays a significant role on controlling rupture.

Wang (2006b) also applied Eqs. (12) and (13) to investigate the shear stress-slip function in a 5 x 5 km square covering a drilled site on the fault plane of the 1999 Chi-Chi, Taiwan, earthquake inferred from seismograms. Results show that the AUD model is more appropriate to describe the inferred shear stress-slip function than the SOP model. He stressed that a more acceptable model is a modified one from the AUD model by including a small amount loss of frictional heat from the slip zone during faulting.

### 4. Scaled Energy

Kanamori (1977) defined the scaled energy as the ratio of seismic radiation energy to seismic moment, i.e., \( e_\sigma = E/M_s \). It can be written as \( (2\Delta\sigma_s - \Delta\sigma_i)/2\mu \) (Kanamori and Heaton 2000). When the stresses fully drop, with \( \Delta\sigma_s = \Delta\sigma_i = \Delta\sigma, \) we have \( E/M_s = \Delta\sigma/2\mu \). The ratio \( E/M_s \) multiplied by \( \mu \) was introduced as the “apparent stress” in seismology (Aki 1966; Wyss and Brune 1968). It can also be written as a product of \( \eta \) and the average stress \( \sigma_0 = (\sigma_s + \sigma_i)/2 \), neither of which can be directly determined seismologically. Either \( \sigma_s \) or \( \sigma_i \), combined with static stress drop, provides useful information for the state of stress in different tectonic provinces. Vassiliou and Kanamori (1982) observed \( E/M_s = 2 \times 10^{-4} \) for shallow earthquake and \( 4.6 \times 10^{-5} \) for deep and intermediate events. For the earthquakes (\( M_s = -1.5 \)) located near the Cajon Pass scientific drill hole, southern California, Abercrombie (1995) observed that \( E/M_s \) increases with magnitude, and \( e_\sigma = 2 \times 10^{-4} \) when \( M_s > 10^{14} \text{ Nm} \) and \( e_\sigma > 2 \times 10^{-4} \) when \( M_s < 10^{14} \text{ Nm} \). Kikuchi and Fukao (1988) observed \( E/M_s = 10^{-6} \text{ - } \times 10^{-5} \), with an average of \( \sim 5.0 \times 10^{-6} \). Izutani and Kanamori (2001) observed an increase in \( E/M_s \) with the earthquake size for 8 earthquakes (\( 3.6 \leq M_s \leq 6.6 \)) in Japan. Kanamori and Heaton (2000) and Prejean and Ellsworth (2001) found an increase in \( E/M_s \) is a function of earthquake magnitude. Large earthquakes (\( M_s > 6 \)) have values of \( \sim 10^{-4} \) while the small ones (\( M_s < 4 \)) have values of \( \sim 10^{-6} \). The transition of \( E/M_s \) occurs almost at \( M_s = 5 \). Brodsky and Kanamori (2001) used an elastohydrodynamic lubrication model to elucidate such a change. Based on the spontaneous rupture model, Ma and Archuleta (2006) theoretically computed the values of \( E/M_s \). For some values of specified model parameters, they obtained \( E/M_s = 6.0 \times 10^{14} \text{ J} \) and \( M_s = 1.47 \times 10^{10} \text{ Nm} \), thus giving \( e_\sigma = 4.08 \times 10^{-5} \).

Theoretically, Kanamori and Rivera (2004) considered that the \( M_s \sim f^{-3} \) scaling relation leads to independence of \( E/M_s \) on earthquake magnitude. They proposed that when the \( M_s \) versus \( f \) scaling is modified from \( M_s \sim f^{-3} \) to \( M_s \sim f^{-\omega} \) (0 < \( \omega \) ≤ 1), the scaled energy can be a function of earthquake magnitude. They also obtained that the optimum value of \( \omega \) is 0.5. Previous observations show that \( \Delta\sigma/2\mu \) is not constant and varies from small events to large ones, thus suggesting that small earthquakes are not similar to large ones. Hence, the GR law which was inferred from large earthquakes cannot be extended to small events.

Wang (2013) studied the correlation of \( e_\sigma \) versus \( M_s \), using two models proposed by Beresnev and Atkinson (1997): (1) the first one is the time function of the average displacements, with an \( \omega^2 \) spectrum, across a fault plane; and (2) the second one is the time function of the average displacements, with an \( \omega^3 \) spectrum, across a fault plane. Model 1 gives independence of \( e_\sigma \) on \( M_s \), and thus the scaled energy is size-independent. This means that in the extreme state of \( E/M_s \), the two different initial conditions, which associated with different source models, lead to the same conclusion that the scaled energy is of size-independence. For Model 2, there are two cases: (1) As \( \tau \approx T \) from a conventional viewpoint, \( \log(e_\sigma) \sim M_s \); and (2) As \( \tau \ll T \) from the slip-pulse concept by Heaton (1990), \( \log(e_\sigma) \sim -M/2 \). Unlike Kanamori and Rivera (2004) and Model 1, Model 2 leads to a negative correlation of scaled energy versus earthquake magnitude. The results obtained from the three different models suggest that the source model, including the scaling law and the relation between \( \tau \) and \( T \), is a factor, yet not a unique one, in controlling the correlation of \( e_\sigma \) versus \( M_s \).

This correlation will depend upon whether the extreme state of \( E/M_s \) is taken into account or not. At present it is not yet known which model is the most appropriate one to explain...
the correlation of $e_R$ versus $M_s$, because the observed correlation is still questionable due to high uncertainties in the estimates of $E_o$, especially for large earthquakes.

On the contrary, Ide and Beroza (2001) used an adjustment factor to account for the probable missing energy, and then observed that $E_o/M_o$ is almost a constant of $\sim 3 \times 10^{-5}$ in a large range of $M_o$ from 4 to 9 or over 17 orders of $M_o$. This value of $E_o/M_o$ is slightly smaller than $5.0 \times 10^{-5}$. For 94 interplate and 74 intraplate earthquakes with $M_o = 10^{15} - 10^{18}$ Nm in the Kanto area, Japan recorded by 27 borehole and 7 surface hard-rock stations, Kinoshita and Ohike (2002) observed $E_o/M_o = (1.15 - 12.9) \times 10^{-5}$ which is weakly dependent on $M_o$. Their average for $E_o/M_o$ is slightly larger than $5.0 \times 10^{-5}$. Yamada et al. (2007) found that the values of $E_o/M_o$ of micro-earthquakes in a gold mine are comparable to those of large earthquakes.

Figure 7 demonstrates the plot of $E_o/M_o$ versus $M_s$ from the data in use (Wang 2015). The horizontal dashed line represents $E_o/M_o = 5.0 \times 10^{-5}$. The data points from Iio (1986) shows a change of $E_o/M_o$ from small events to large ones at $M_s = 2.5$, which is smaller than the transition magnitude ($M_s = 5$) proposed by Brodsky and Kanamori (2001). This might be due to under-estimates of $E_o$ for micro-events by Iio (1986). The overall distribution of $E_o/M_o$ versus $M_s$ is quite uniform and around the horizontal dashed line with $E_o/M_o = 5.0 \times 10^{-5}$ and there is not a transition at $M_s = 5$. Obviously, $E_o/M_o$ is approximately a constant for the present data.

From the strong-motion seismograms recorded by the SMART-1 array generated by 21 near-earthquakes ($M_s = 4.1$ - 7.8) with focal depths from 1 - 98 km, Bolt and Wen (1990) measured the values of $M_s$. The measured values are $M_s = 6.0 \times 10^{25} - 1.3 \times 10^{27}$ dyne·cm. From their measured values of $E_o$, as mentioned above, they obtained $E_o/M_o = (4.25 \pm 0.12) \times 10^{-4}$, which is about one-order magnitude larger than the common value of $5.0 \times 10^{-5}$. This might be due to over-estimates of $E_o$, because they did not eliminate the FFBL and site effects.

From teleseismic data the $e_R$ value for the Chi-Chi earthquake evaluated by Venkataraman and Kanamori (2004) is $2 \times 10^{-5}$. Huang et al. (2002) and Huang and Wang (2009) measured the $E_o$ and $M_s$ values of twenty-two larger-sized aftershocks with ($4.4 \leq M_s \leq 6.5$) from the 1999 Chi-Chi, Taiwan, earthquake from local seismograms. Results are: $E_o = 2.0 \times 10^{18} - 8.9 \times 10^{21}$ dyne·cm and $M_s = 1.3 \times 10^{23} - 1.4 \times 10^{20}$ cm·dyne, thus leading to $E_o/M_o = 7.4 \times 10^{-6} - 2.6 \times 10^{-4}$. The $E_o/M_s$ values of the 22 events are dependent upon $M_s$. They also used the $M_o$ values measured from teleseismic data to calculate $E_o/M_s$. The results show $E_o/M_s$ independence from $M_s$ when teleseismic $M_o$ values are used. They also measured the corner frequency, $f_c$. Their results ranged from 0.15 - 1.34. The scaling law between $M_s$ and $f_c$ is $M_s \sim f_c^{-3.65}$.

From the measured values of $E_o$ and $M_s$ by Hwang (2012) for the M 6.4 JiaSian earthquake of 10 March 2010 as mentioned above, the scaled energy of the event is $E_o/M_o = 1.3 \times 10^{-5}$, which is lower than ordinary earthquakes.

5. ENERGY-MAGNITUDE LAW

Richter (1935) defined the local magnitude, $M_l$, from seismograms recorded on the standard Wood-Anderson seismograph. Gutenberg (1945) defined the longer-period body-wave magnitude, $m_b$, and surface-wave magnitude, $M_{gr}$. $M_{gr}$ is measured from the maximum ground horizontal surface-wave amplitude, i.e., $A = (A_N^2 + A_E^2)^{1/2} = \sqrt{2}A_N$ or $\sqrt{2}A_E$, where $A_N$ and $A_E$ are the maximum ground horizontal surface-wave amplitudes along the E-W and N-S directions, respectively, recorded at an epicentral distance of $\Delta = 15 - 130^\circ$. The wave period in use is $T = 17 - 23$ sec.

The relationship between the seismic radiation energy and earthquake magnitude is important in quantifying earthquakes. Using strong-motion data from 18 California earthquakes with $3.9 \leq M_s \leq 7.3$, Gutenberg and Richter (1956) inferred a relationship between $E_o$ and $M_s$ in the following form: $\log(E_o) = 12 + 1.8M_{gr}$ (in erg). Bullen (1955) expressed that Gutenberg and Richter determined a new relationship: $\log(E_o) = 11 + 1.6M_{gr}$ (in erg). Gutenberg and Richter (1956) reported that the previous $E_o$ - $M_s$ relationship is wrong due to over-estimates of $E_o$. They found a new law: $\log(E_o) = 5.8 + 2.4m_b$ (in erg). Based on this relationship: $m_b = 2.50 + 0.63M_{gr}$, they obtained $\log(E_o) = 11.8 + 1.5M_{gr}$ in erg or $\log(E_o) = 4.8 + 1.5M_{gr}$ (in joule) from the data with $M_{gr} \geq 5.5$.

Vanek et al. (1962) defined a new surface-wave magnitude, that is, $M_s = \log(\Delta/A) + 1.66\log(\Delta) + 3.3$. $M_s$ measures the maximum ground vertical surface-wave amplitude, i.e., $A_v$, recorded at an epicentral distance of $\Delta = 20 - 160^\circ$ and focal depth less than 50 km for the USGS and $\Delta = 20 - 160^\circ$
Can $M_{GR}$ be replaced by $M$ in the GR law? Abe (1981) found the equivalence of $M_i$ to $M_{GR}$ for global earthquakes. Lienkaemper (1984) observed that $M_i$ for the same events recomputed with the Prague formula is only 0.03 units of $M_i$ higher on average than $M_{GR}$. Wang and Miyamura (1990) and Wang (1992) found the similarity of the two magnitude scales for Taiwan’s earthquakes. Based on the earthquake source spectra proposed by Aki (1967), the wave amplitudes with $T = 18 - 22$ sec are almost the same as those with $T = 17 - 23$ sec. Consequently, $M_{GR}$ can be replaced by $M_i$ in the GR law.

The scaling exponent of the GR law is 1.5. Theoretically, seismic radiation energy generated from a dynamic crack with an area of $A$ is $E_o = \Delta\sigma DA/2$, where $\Delta\sigma$ and $D$ are, respectively, the static stress drop and average slip on the crack plane, when the stress fully drops and the fracture energy can be negligible. This leads to $E_o = \Delta\sigma M/2\mu$ and thus $\log(E_o) = \log(\Delta\sigma/2\mu) + \log(M_i)$ where $M_i = \mu DA$ is the seismic moment (Aki 1966; Aki and Richards 1980). Purcaru and Berckhemer (1978) obtained a scaling relationship between $M_i$ and $M_o$, i.e., $\log(M_o) = 1.5M_i + 16.1$ (in dyne cm). This yields $\log(E_o) = 11.8 + 1.5M_i$, under $\Delta\sigma/2\mu = 5.0 \times 10^{-3}$ which is a common value for most earthquakes (Knopoff 1958; Kanamori 1977). It is obvious that the $E_o - M_i$ relationship obtained from the dynamic crack model, together with an empirical relationship between $M_i$ and $M_o$, is the same as the GR law. This is the physical basis for making the GR law valid.

Since Gutenberg and Richter (1956) presented the GR law, numerous $E_i - M_i$ relationships have been inferred by various authors from different data sets. Some examples are given below. From teleseismic data, there are $\log(E_o) = 7.75 + 1.87M_{GR}$ by Tocher (1958); $\log(E_o) = 7.2 + 2M_{GR}$ by Bath and Duda (1964); and $\log(E_o) = 7.5 + 2M_{GR}$ by Reid et al. (cf. Fiedler 1967). From strong-motion seismograms, Vassiliou and Kanamori (1982) obtained $\log(E_o) = (9.06 \pm 1.38) + (1.81 \pm 0.20)M_i$ ($M_i \geq 5.9$). From the strong-motion seismograms recorded by the SMART-1 array generated by 21 near-earthquakes ($M_i = 4.1 - 7.8$) with focal depths from 1 - 98 km, Bolt and Wen (1990) measured the $E_o$ values of those events using the integral of the square of the ground velocity. The measured values are $E_o = 8.612 \times 10^{18} - 1.183 \times 10^{23}$ g cm$^2$ sec$^{-2}$. They inferred a relationship between $E_o$ and $M_i$; $\log(E_o) = (14.71 \pm 1.06) + (1.12 \pm 0.19)M_i$, Pérez-Campos and Beroza (2001) inferred numerous $E_i - \log(M_i)$ relationships. Obviously, these relationships are in general different from the GR law and the $E_o$ value calculated from their formula is higher than that from the GR law when $M_i < 7.7$. This might be due to over-estimates of $E_o$ by them, because the site effect was not removed from their measures, especially for strong-motion data. In addition, Kikuchi and Fukao (1988) observed that the $E_o$ values of thirty-five $M_i > 6$ earthquakes are smaller than those calculated from the GR law.

Choy and Boatwright (1995) compiled a data set of $397$ global events with $M_i > 4.4$. For the events with $M_i > 5.7$, they inferred an $E_i - M_i$ relationship: $\log(E_i) = 11.4 + 1.5M_i$ (in ergs) or $\log(E_i) = 4.4 + 1.5M_i$ (in J). This $E_i - M_i$ relationship is slightly different from the GR law. They also assumed that the GR law slightly over-estimates $E_i$. Nevertheless, the scaling exponents of the two laws are both 1.5.

Only the earthquakes with $M_i > 5.5$ were taken by Gutenberg and Richter (1956) and Choy and Boatwright (1995) to infer the $E_i - M_i$ scaling law. Hence, in principle the GR law can be applied to evaluate $E_i$ only for $M_i > 5.5$ earthquakes. However, the GR law has also been applied to evaluate $E_i$ even for small earthquakes for a long time. Naturally, a question appears: Can the law be applied to evaluate $E_o$ for micro- and small events with $0 \leq M_i \leq 5.5$?

In order to answer this question, the $E_o - M_i$ relationship for earthquakes with $M_i \leq 5.5$ must be studied from related $E_o$ and $M_i$ values for earthquakes with $M_i \leq 5.5$. Several groups of researchers (e.g., Iio 1986; Choy and Boatwright 1995; Izutani and Kanamori 2001; Yamada et al. 2007; Huang and Wang 2009; Sivaram et al. 2013) measured the $E_o$ values for micro- and small earthquakes with $M_i \leq 5.5$ in different regions. One hundred sixty-six events, with $0.0 \leq M_i \leq 5.5$, $9.0 \times 10^6$ Nm $\leq M_i \leq 1.2 \times 10^8$ Nm, and $1.28 \times 10^3 J \leq E_o \leq 3.30 \times 10^{14} J$, occurring in different regions were collected by Wang (2015) to study the problem. Except for the events with $0.5 < M_i < 1.5$ from Iio (1986), the data points of $E_o$ versus $M_i$ for earthquakes with $M_i \leq 5.5$ are almost around the GR law. This suggests that the seismic radiation energy of earthquakes with $M_i \leq 5.5$ can be evaluated from the GR law. Due to scattering of data points it is not easy to be sure if the GR law is better than the $E_o - M_i$ relationship inferred by Choy and Boatwright (1995) to interpret the observed data or not.

6. SUMMARY

Seismic radiation energy studies made pre-1999 are reviewed in this work. For those using the GR law to calculate $E_o$, the results should be corrected due to the use of non-$M_i$ earthquake magnitudes. The measure results for $E_o$ from strong-motion seismograms are questionable because the authors did not remove the FFBL and site effects. The theoretical studies on the FFBL effect by Wang (2004) and Wang and Huang (2007) and the site effect by Huang et al. (2005, 2007, 2009) are described and discussed in this review article.

For the $M_i 7.6$ Chi-Chi earthquake of 20 September 1999, which ruptured the Chelungpu fault in Central Taiwan,
Wang (2004, 2006b) measured the strain energy (\(\Delta E\)), seismic radiation energy (\(E_r\)), fracture energy (\(E_f\)), and frictional energy (\(E_g\)) for the whole fault and its two segments. There are differences in the four kinds of energy between the northern and southern segments. Several important concluding points are given below:

1. Wang (2004) obtained \(\Delta E = 3.206 \times 10^{23}\) ergs, \(\Delta E_n = 2.341 \times 10^{23}\) ergs, and \(\Delta E_s = 0.865 \times 10^{23}\) ergs, respectively, for the northern and southern segments, respectively.

2. From near-field seismograms, Wang (2004) obtained \(E_r = 4.307 \times 10^{23}\) ergs and \(E_{r_n} = 3.981 \times 10^{23}\) ergs for the northern segment and \(E_{r_s} = 0.326 \times 10^{23}\) ergs for the southern segments. The seismic efficiency, \(\eta = E_r/\Delta E\), of the earthquake obtained from \(E_r\) measured from near-field seismograms and that from teleseismic data are, respectively, 0.137 and 0.262, which do not agree with the \(\eta \leq 0.06\) hypothesis proposed by McGarr (1994). On the other hand, based on the \(E_r\) calculated from two \(E_r-M\) laws, \(\eta = 0.049\) and 0.019, which fit McGarr’s hypothesis. However, the two laws could underestimate \(E_r\). The radiation efficiency, \(\eta_r = E_r/(E_r + E_g)\), are also evaluated.

3. From local seismograms, Wang (2006b) obtained the optimum values: (1) \(E_r = 0.15 \times 10^6\) J, \(\eta_r = 0.69\), and \(G = 0.33 \times 10^7\) J m\(^{-2}\) for the southern segment; and (2) \(E_r = 1.99 \times 10^6\) J, \(\eta_r = 0.67\), and \(G = 5.34 \times 10^7\) J m\(^{-2}\) for the northern segment.

4. For the frictional heat, \(E_g\), caused by dynamic frictional stress, there is a marked difference between the two segments. The average frictional and ambient stress levels on the two segments are estimated. The total energy budget of and heat generated by the earthquake are elucidated based on a 2-D faulting model with frictional heat. Both observed and calculated results suggest the possible existence of fluids, which produced suprahdrostatic gradients, on the fault during faulting. Lubrication and thermal fluid pressurization might play a significant role on rupture.

5. From the core samples obtained at Hole A of the TCDP, Wang (2011) evaluated the heat strength (= 7.0°C m), within a heated layer of ~5 mm, due to frictional faulting from the values of shear stress and thermal and mechanical parameters. Based on a 1-D heat conduction equation and 2-D faulting model, with the values of thermal diffusivity evaluated within a representative temperature range, the thermal and pore fluid pressure history at depths 1110.37 - 1111.34 m in Hole-A is constructed. Results show that the peak temperature at the center of the heated layer could have been higher than 1100°C during faulting, and the temperature rise decreased quickly with increasing distance and time. This provides an answer to the so-called heat flux paradox (Lachenbruch and Sass 1980). There is no high thermal anomaly during faulting due to a remarkable decrease in the effective frictional stress.

(6) There are remarkable relationships between the temperatures and chemical reactions of clay minerals. In the heated layer, pseudotachylites have been formed and quartz plasticity might also have been operative during faulting. Outside this slip zone, the temperature rise was low and thus clay minerals were stable during faulting. The evaluated pore fluid pressure is 22.5 MPa, thus suggesting the existence of a suprahdrostatic state in the fault zone during the earthquake.

The radiation efficiency, \(\eta_r\), is strongly affected by the variation in shear stress with slip. Wang (2009) considered thermal pressurization to be a significant mechanism in controlling such a variation, thus influencing \(\eta_r\). He derived the formulae of \(\eta_r\) as a function of slip, \(\delta\), on the basis of two end-member models of thermal pressurization, i.e., the AUD and SOP models, proposed by Rice (2006) are derived. The controlling parameters of the AUD and SOP models are, respectively, \(\delta\) and \(L^*\) which are individual in terms of the thermal, mechanical, and hydraulic parameters of fault rocks. Modeled results show that \(\eta_r\) increases with \(\delta\). The increasing rate of \(\eta_r\) with \(\delta\) is high at small \(\delta\) and low at large \(\delta\). This indicates that \(\eta_r\) varies very much with \(\delta\) for small earthquakes and only slightly depends on \(\delta\) for large events. For the two end-member models, \(\eta_r\) increases with decreasing \(\delta\) (or \(L^*\)). When \(\delta = L^*\), \(\eta_r\) is higher for the AUD model than for the SOP model.

The correlation of the scaled energy, \(e_r = E_r/M_s\), versus earthquake magnitude, \(M_s\), is studied based on two models: (1) Model 1 based on the \(\omega^2\) source model; and (2) Model 2 based on the \(\omega^3\) source model. The results show that Model 1 influences the correlation of \(e_r\) versus \(M_s\); the source model is a factor, yet not a unique one, in controlling the correlation; and Model 2 cannot work for studying this correlation.

The scaling law of \(E_r\) versus \(M_s\), proposed by Gutenberg and Richter (1956) was originally based on \(M_s > 5.5\) earthquakes. Wang (2015) found that this law is also valid for earthquakes with \(0 < M_s < 5.5\). Meanwhile, the scaled energy is almost constant for \(M_s > 0\) earthquakes.

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