# Simplified Nonhyperbolic Moveout Formula 

# for a Transversely Isotropic Medium 

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#### Abstract

A new methodology of skew hyperbolic moveout analysis has been proposed to obtain the velocity model of reflection trajectories through an anisotropic medium. The original skew formula describes the trajectory in three velocity parameters. Conformation from the skew hyperbolic analysis suggested that one of the velocity parameters in the skew formula can be omitted/replaced. Furthermore, the moveout curve of nonhyperbolic analysis could also be analyzed by using only two velocity parameters. With one less parameter, we, therefore, call the modified skew formula a simplified nonhyperbolic moveout formula.

To ascertain the practicality of our new formula, experiments were done through an isotropic (plexiglas) block and an anisotropic (phenolite) block. A semblance analysis using the simplified formula was then applied to scan the recordings. For transverse isotropy of periodic interleaving of isotropic layers, the moveout velocity as deriven from reflection seismology contains vertical and horizontal components. The vertical component can be applied to perform time-to-depth conversion, and the anisotropy sensitive velocity ratio of horizontal-to-vertical can be used to investigate lithology or physical properties of the subsurface datum.


(Key words: Moveout analysis, TIM)

## 1. INTRODUCTION

In reflection seismology, the velocity information of explored datum is commonly obtained indirectly by time-distance relationships analysis through reflection recordings acquired by surface seismic measurements. For a homogenous and isoropic stratum, under short spread

[^0]case, the reflection rajectories from a common shotpoint gather (CSG) or a common midpoint gather (CMG) follows a hyperbolic curve, this analyzing technique is referred as hyperbola analysis (Gardner 1947; Slotnik 1959). However, because the computed velocity, $\mathrm{V}_{\text {mmo }}$, is the vertical velocity of the explored strata, the hyperbola analysis thus becomes inappropriate for multilayered cases since rays would bent at each interface. Dix (1955) adopted the idea of least-time path proposed the concept of the root-mean-square velocity, $\mathrm{V}_{\mathrm{rm}}$, in order to resolve small offset. The merit of the $\mathrm{V}_{\mathrm{ms}}$ concept is that the interval velocity of an individual layer is measurable from the $\mathrm{V}_{\mathrm{ms}}$ distribution. The concept of root-mean-square velocity was then extended to layers with dipping by Levin (1971) and Shah (1973). The clarity of $\mathrm{V}_{\mathrm{ms}}$ analysis contributed its popularity and wide usage. However, for widespread field surveys or in the presence of anisotropy media, Dix's formula may be invalid.

Byun et al. (1989), and Tsvankin and Thomsen (1994) had demonstrated the trajectory of reflection moveout curve on a CSG or CMG no longer follow a hyperbolic curve for wide survey range or in the presence of anisotropy. To obtain more reliable velocity information from reflections, Byun et al. (1989) derived a new formula, the skewed hyperbolic moveout formula, to perform velocity analysis for horizontally layered, transversely isotropic media. With the skew formula, three velocity parameters: the average vertical velocity $\left(V_{v}\right)$, horizon$\operatorname{tal}\left(V_{h}\right)$ and skew $\left(V_{\gamma}\right)$ moveout velocities can be derived. Unless the raypathes of the seismic energy falls on a sagittal plane of isovelocity, the reflection trajectories described by skew formula may not be a hyperbola one, making Byun's formula a nonhyperbolic analysis. Carrying on Byun's work, Sena (1991) derived traveltime-offset curves from analytical expressions of multilayered weakly azimuthally isorropic and anisotropic media with elastic properties. The nonhyperbolically analysis done on reflections provided a new way to access the elastic properties of subsurface datum from surface reflections. To simplify the process of skew formula, we demonstrate the reflections can be well described by a modified skew formula for a transverse isotropic medium composed of periodic interleaving isotropic layers, which are thin compared to the predominant wavelength of the explored waves. Because the modified skew formula contains only two velocity parameters, we referred the modified formula as a simplified nonhyperbolic moveout formula. Laboratory trials were carried out to confirm the practicality and accuracy of this simplified nonhyperbolic formula.

## 2. REVIEW OF THE MOVEOUT ANALYSIS

### 2.1 Hyperbola Moveout Analysis

The success of an exploration job depends greatly on the velocity information derived. With reliable velocity, seismic data can be efficiently processed and confidently interpreted. In exploration history, many efforts have been contributed to derive the velocity information of subsurface strata based on different models. For a single horizontal and isotropic layer model (Fig. 1), using the Pythagorean theorem, the traveltime of a reflection signal can be related to the offset distance between source and receiver by the following equation:

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Fig. 1. The ray geometry for a CMG shooting. $z$ is reflection depth, and $x_{\mathrm{S}}$ and $x_{\mathrm{R}}$ are the source and receiver positions, respectively.

$$
\begin{equation*}
t_{x}^{2}=t_{0}^{2}+\frac{x^{2}}{V_{n m o}^{2}} \tag{1}
\end{equation*}
$$

In equation (1), $\mathrm{t}_{\mathrm{x}}$ is the traveltime of a reflection along the raypath SMR in Fig. 1; $\mathrm{to}_{0}$ is the two-way vertical (zero-offset) arrival time; $x=\left(x_{R}-x_{S}\right)$ and is the offset distance between source ( S ) and receiver ( R ); and $\mathrm{V}_{\mathrm{nmo}}=z /$ to is called the "moveout velocity". Mathematically, equation (1) represents a hyperbolic trajectory. The $\mathrm{V}_{\mathrm{nmo}}$, deriven from analyzing the trajectory of reflections is, called hyperbola analysis and presents the layering velocity of the datum.

For the n-layer (multilayered) case, under the assumption of short survey length, Dix (1955) showed the effect of ray bending due to the existence of interface could be overcomed by replacing the $\mathrm{V}_{\mathrm{nmo}}$ in equation (1) with the root-mean-square velocity, $\mathrm{V}_{\mathrm{rms}}$ :

$$
\begin{equation*}
V_{r m s}=\left[\frac{\sum_{i=1}^{i=n} V_{i}^{2} t_{i}}{\sum_{i=1}^{i=n} t_{i}}\right]^{1 / 2} \tag{2}
\end{equation*}
$$

where $t_{i}$ is the two-way traveltime on a vertical path through the layer of velocity $V_{i}$ that refers to a specific raypath. The contribution of the $\mathrm{V}_{\mathrm{rms}}$ is that the interval velocity of the stratum to be explored is measurable from the $\mathrm{V}_{\mathrm{rms}}$ distribution by subtracting equation (2) for the RMS velocity of the ( $\mathrm{n}-1$ )th-layer from the RMS velocity of the nth-layer case. The assumption of short spread made by Dix was then released by Brown (1969), Taner and Koehler (1969), and

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Shah and Levin (1973). Shah (1973) extended the concept of RMS velocity to dipping layers with arbitrary and nonuniform dips.

### 2.2 Skew Hyperbolic Moveout Analysis

Using a higher order polynomial fit methodology, Byun et al. (1989) and Sena (1991) proposed the idea of "skewed hyperbolic fitting". For a horizontal but transversely isotropic layered subsurface, both of them showed that the rajectory of reflections could be better specified by their skewed hyperbolic formula than the simple hyperbolic analysis of equation (1). Considering the ray geometry depicted in Fig. 1, the following equation shows the skewed hyperbola moveout formula derived by Byun et al:

$$
\begin{equation*}
t_{x}^{2}=\left(\frac{2 z}{V_{v}}\right)^{2}+\left[\left(\frac{z}{V_{\gamma}}\right)^{2}+\left(\frac{x}{2 V_{h}}\right)^{2}\right] \frac{x^{2}}{z^{2}+(x / 2)^{2}} \tag{3}
\end{equation*}
$$

Their new formula involves three measurement parameters: the average vertical velocity $\left(V_{v}\right)$, horizontal $\left(V_{h}\right)$ and skew moveout velocities $\left(V_{\gamma}\right)$. For azimuthal isotropic media (VTI), velocities $V_{v}, V_{h}$, and $V_{\gamma}$, in equation (3) are expressed as:

$$
\begin{align*}
V_{v}^{2} & =\alpha_{0}^{2}=\frac{C_{33}}{\rho}  \tag{4}\\
V_{\gamma}^{2} & =\alpha_{0}^{2}(1+2 \delta)  \tag{5}\\
V_{h}^{2} & =\alpha_{0}^{2}(1+2 \varepsilon)=\frac{C_{11}}{\rho} \tag{6}
\end{align*}
$$

For azimuthal anisotropic media (HTI), velocities, $V_{v}, V_{h}$, and $V_{\gamma}$, become (Sena 1991):

$$
\begin{align*}
& V_{v}^{2}=\frac{C_{11}}{\rho}  \tag{7}\\
& V_{\gamma}^{2}=\alpha_{0}^{2}\left[1+2 \varepsilon \sin ^{2}(\varphi-\phi)-2(\varepsilon-\delta) \cos ^{2}(\varphi-\phi)\right]  \tag{8}\\
& V_{h}^{2}=\alpha_{0}^{2}\left\{1+2 \sin ^{2}(\varphi-\phi)\left[\varepsilon \sin ^{2}(\varphi-\phi)+\delta \cos ^{2}(\varphi-\phi)\right]\right\} . \tag{9}
\end{align*}
$$

$\mathrm{C}_{11}$ and $\mathrm{C}_{33}$ are two of five elastic constants that are commonly used in the specification of elastic properties of the TIM. The anisotropic parameters: $\varepsilon \& \delta$, used in equations 4 to 9 follow Thomsen's (1986) definition. Notations $\varphi$ and $\phi$ shown in equations 8 and 9 stand for

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the azimuths of survey line and of the horizontal axis of symmetry of a TIM to the reference coordinate, respectively. For an isotropic case, i.e. $V_{v}=V_{h}=V_{\gamma}$, equation (3) becomes the form of equation (1).

### 2.3. Simplified Nonhyperbolic Moveout Analysis

The use of skew hyperbolic analysis to process seismic reflections provide a new aspect to analyze the moveout velocities. Physically, velocity involves both direction and magnitude, and the actual velocity is a vector whose magnitude depends upon its direction. Therefore, any process that involves the concept of a scalar velocity field will be subject to error. Assuming that the raypath falls on a sagittal plane, we compile a simplified nonhyperbolic moveout formula. Considering the ray geometry shown in Fig. 1, the simplified formula is expressed as:

$$
\begin{equation*}
\left(t_{S R}\right)^{2}=\left(2 t_{S M}\right)^{2}=\frac{(2 z)^{2}}{V_{v}^{2}}+\frac{(x)^{2}}{V_{h}^{2}}=t_{0}^{2}+\frac{x^{2}}{V_{h}^{2}} \tag{10}
\end{equation*}
$$

where $t_{s R}$ is the total traveltime for a reflection signal and $t_{S M}$ is the traveltime from source to reflection point (M). In the simplified formula (equation 10), the seismic velocity is now decomposed into two measurement components: vertical velocity $\left(V_{v}\right)$ and horizontal velocity $\left(V_{h}\right)$.

For zero-offset reflection, i.e. $\mathrm{x}=0$, the vertical velocity, which is commonly used to estimate the thickness of the explored sedimentary layer, can be easily obtained. For intermediate spread ( $1.5 \leq x / z \leq 2.5$ ) cases, set $2 z / V_{v}=\mathrm{t}_{0}$, equation (10) takes the form of equaion (1). The moveout velocity, $\mathrm{V}_{\mathrm{nmo}}$, in equation (1) is now replaced by the horizontal velocity, $V_{h}$. Equation (10) indicates that the velocity obtained from the traveltime equation thus essentially represents the horizontal velocity of the layer.

## 3. LABORATORY WORK

To see how the equation (10) would work for analyzing velocity from CSG or CMG reflection, laboratory experiments were collected and studied. For comparison, two media of plexiglas and phenolite were utilized. Plexiglas is a transparent block with 3.9 cm in thickness made of resin, and is considered as an isotropic material. Phenolite is a transversely isotropic material, made of layers of paper bonded with phenolitic resin with 4.0 cm in thickness. The intrinsic properties, i.e. interlacing of paper and resin, can be used to simulate sedimentary strata, systematic fractures, minerals with preferred alignment, etc., thus making it popular in laboratory work. Moreover, for a phenolitic block, the transmission observations were done on vertical transversely isotropic (VTI) model with vertical symmetry axis, and horizontal transversely isotropic (HTI) model with horizontally symmetry axis. Some physical properties for both plexiglas and phenolitic block is shown in table 1.

For zero-offset reflection, seismic energy travels vertically downward at the shot point, and is reflected vertically upward. The traveltime thus can be easily computed by dividing
two-times of layer thickness by the velocity. Two times the thickness can be considered as a transmission shot done on a layer of double thickness. With similar idea, a transmission experiment was done on our models. The data collected, therefore, can be treated as reflections from the same model of one half in thickness. The arrangement of experimental setup is shown in Fig. 2. For transmission experiment, two P-type of Panametric ${ }^{\mathrm{TM}}$ transducers of 6 mm in diameter and 2.25 MHz manufactured frequency were placed face to face on opposite side of a modeling block. The experiments started with the source transducer placed directly beneath the receiver. In the face-to-face case, direct arrival of the acoustic energy is measured and can be considered as a zero-offset reflection from the same model with one half thickness. In the process of data collection, the source transducer was fixed at its original position with the receiver successively moved in increment of $0.25-\mathrm{cm}$. The sampling interval used for the laboratory operation was 40 microseconds, and the scaling factor applied during the process of data acquisition was 10,000 .

The dimension of the model limits the survey length. Therefore, for the isotropic model (plexiglas) and the VTI model, we collected 21 traces of $5-\mathrm{cm}$ surveying length. For the HTI model, we collected 13 traces of $3-\mathrm{cm}$ surveying length. For reflection seismology, the ratio of survey length to thickness $(x / z)$ is 2.5 for isotropic and VTI models, and is 1.5 for the HTI model. Indicated our measurements belong to the intermediate-to-long spread reflection catagory(Tsvankin and Thomsen 1994), and provide essential requirements to inspect the moveout differences (Li 1999).

## 4. DATA ANALYSIS

Results of laboratory observation are shown in Figs. 3 (isotropic model), 4 (VTI model), and 6 (HTI model). Trace 1 in Figs. 3, 4 and 6 represents a zero-offset reflection. In processing the laboratory data, the trajectory of the reflections were scanned by both hyperbola formula and simplified nonhyperbolic moveout formula. The short dashed lines in Figs. 3, 4 and 6 describe the trajectory of the best fitting curves obtained by hyperbola analysis Equation 1, while solid curves and long dashed curves were computed by simplified nonhyperbolic formula (Equation 10). For solid curves, scanning was done on all traces. Constant offset between shot point and nearest geophone station is present in field survey. To meet the field condition, the long dashed curves were obtained by scanning the traces of each gather after trace 5. The $\mathrm{V}_{\text {hyp }}$ shown in table 2 is the $\mathrm{V}_{\text {nmo }}$ in equation (1). The computed $V_{v}$ and $V_{h}$ for zero offset and near offset fitting are also shown in table 2.

For plexiglas, all fitting curves: hyperbola, zero offset, and short offset fittings almost run along the same trajectory in Fig. 3. Results indicated that the derived velocities from moveout equations 1 and 10 are not obviously wavy, i.e. $V_{\text {hyp }} \approx V_{v} \approx V_{h}$. For phenolite, the anisotropic case, the poor match of short dashed curves to the reflections in VTI and HTI models exhibits the intrinsic effects of anisotropy velocity through the explored medium. For VTI model, the hyperbolic fitting curve in Fig. 4 does not describe the trajectory of the reflections well. However, the other two nonhyperbola fitting curves almost run along the same trajectory.

For HTI model, the survey lines run along three azimuths: layering, diagonal, and axis of

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Table 1. The velocities and densities of the Plexiglas and phenolite used in the experiments. $\perp$ represents the velocity measured along the symmetry axis; $/ /$ stands for the velocity measured along the layering of the phenolitic block.

| Modeling material | Density $\left(\mathrm{gm} / \mathrm{cm}^{3}\right)$ | P-wave velocity $(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: |
| Plexiglas | 1.1 | 2,876 |
| Phenolite | 1.4 | $2,950(\perp)$ |



Fig. 2. Depiction of laboratory setup for transmission experiment. (a) VTI model and (b) HTI model. The thickness of modeling block is $4-\mathrm{cm}$. Offset interval for successive receiver is $0.25-\mathrm{cm}$.

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Fig. 3. Transmission data for isotropic model. In this figure and Figs. 4 and 6, the short broken line is a hyperbolic fitting curve, solid line is nonhyperbolic fitting curve for zero offset, and long broken line is short offset fitting curve. All of three fitting curves are almost tie in the figure for isotropic case.


Fig. 4. Transmission for VTI model. It can be seen that the hyperbola fitting does not describe the trajectory of reflections well; however, nonhyperbolic fittings do.

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symmetry (Fig. 5). Results of observation were shown in Figs. 6a, b, c. In Fig. 6a, the transmission experiment was done along the layering. For the TIM model, the velocities of compressive ( P -) waves along layers do not vary with the propagation direction. Results show all of the three fitting curves are well matched (Fig. 6a), which is similar to that in isotropic case (Fig. 2). For P -wave reflection exploration, the measurements done along layerings thus can be considered as a special case for the isotropic one. In Fig. 6b, the observation was done along the azimuthal angle of 45 degrees to the layering and the hyperbolically computed curve does not ideally fit the reflections. Yet the other two nonhyperbolically fitted curves exhibit good agreement with the reflections. In Fig. 6c, the observation was done along the symmetry axis of the HTI model. Again, instead of hyperbola fitting, the nonhyperbola fitting curves better describe the rajectories. The deviation between the hyperbolic and nonhyperbolic fitting curves become obvious in Fig. 6c.

## 5. DISCUSSION

For reflection seismology, the traveltime is a function of source-receiver offset and velocities of the subsurface strata by which the seismic energy propagated. The traveltime equa-

Table 2. Results of nonhyperbolic fitting. In the laboratory trials, semblance analysis was performed along the $t-x$ curves by scanning over ranges of the vertical $\left(V_{\nu}\right)$ and horizontal $\left(V_{h}\right)$ velocities. For short spread fitting, the analysis was applied to recorded traces whose offset exceed $1-\mathrm{cm}$, i.e. traces after trace 5. For zero-offset fitting, the semblance analysis was applied to all recorded traces.

|  | Hyperbola Fitting | Nonhyperbola Fitting |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Isotropic Fitting <br> $V_{h y p}(\mathrm{~m} / \mathrm{s})$ | Short Offeet Fitting |  | Zero Spread Fitting |  |
|  | $-V_{v}(\mathrm{~m} / \mathrm{s})$ | $V_{h}(\mathrm{~m} / \mathrm{s})$ | $V_{v}(\mathrm{~m} / \mathrm{s})$ | $V_{h}(\mathrm{~m} / \mathrm{s})$ |  |
| Plexiglas | 2,876 | 2,903 | 2,835 | 2,903 | 2,835 |
| Phenolite <br> $($ VTI) | 3,096 | 2,824 | 3,836 | 2,867 | 3,726 |
| Phenolite <br> (HTI; //) | 3,948 | 4,005 | 3,920 | 4,000 | 3,945 |
| Phenolite <br> (HTI; 45) | 3,949 | 4,109 | 3,446 | 4,047 | 3,546 |
| Phenolite <br> (HTI; $\perp$ ) | 3,840 | 4,072 | 3,046 | 4,037 | 3,095 |

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Fig. 5. Top view of survey line arrangement for HTI model. Line I runs along the layering direction, Line III runs along the symmery axis, and Line II runs diagonal to both layering and symmetry axis. The source transducer was positioned directly beneath the interception point of the survey lines. Results of observation are shown in Fig. 6.
a

b


C


Fig. 6. Transmission data for HTI model. The measurements were done along the layering (Line I) for (a), along the angle of azimuth 45 degrees to both layering and symmetry axis (Line II) for (b), and along the symmetry axis for (c).
tion is therefore expressed as a function of traveling distance and velocities of the explored strata. Equations that will be applied to perform velocity analysis should describe the trajectory of the reflections well. In the application of the traditional hyperbola analysis, Yilmaz (1987) pointed out the derived velocity is highly dominated by the spread length, and the reflection trajectories do not follow a perfect hyperbola. In addition to the spread length, the anisotropy of the exploring medium could cause the trajectory of the $t-x$ curve to deviate (Levin 1979; Hake et al. 1984; Byun et al. 1989; Tsvankin and Thomsen 1994).

Taking anisotropy into consideration, Bynn et al. (1989) showed that the $t-x$ curve of the reflections can be well described by a new formula, called skew hyperbolic formula. In the skew formula, the traveltime equation is a function of traveling distance with three velocity parameters: $V_{v}, V_{h}$ and $V_{\gamma}$. Among these three velocity parameters, the $V_{\gamma}$ is dominated by the "strange" coefficient, $\delta$. For a transversely isotropic medium made of periodic interleaving of isotropic layers, Banik (1984) showed that $\delta$ is null as the velocity ratio is constant across all layers. By examing equations 5 and 8 , it can be found that $V_{\gamma}$ can possibly be replaced by $V_{v}$. Once $\delta$ is set to zero, the $V_{\gamma}$ formulated by equation (5) becomes $V_{v}$ of equation (4) for a VTI model. For an HTI model $V_{\gamma}$ (equation 8) comes to the $V_{v}$ (equation 7) if both $\varphi$ and $\phi$ are zero. The foregoing discussion provides us reason to rewrite the skew formula into a similar form with one less velocity parameter. The rewritten form of skew formula is named as simplified nonhyperbolic moveout formula (equation 10), in which the number of velocity parameter is reduced from three to two.

Velocities $\mathrm{V}_{\mathrm{hyp}}, V_{v}$ and $V_{h}$, shown in table 2 are computed from our laboratory data by hyperbolic fitting (equation 1), and simplified nonhyperbolic moveout formula (equation 10 ). For the isotropic model, plexiglas, the computed velocities $\mathrm{V}_{\mathrm{hyp}}, V_{v}$, and $V_{h}$, are almost equal. The good coincidence of all three fitting curves shown in Figs. 3 and 6a shows that the hyperbola velocity analysis works only under limited situation. For anisotropic models, the hyperbola fitting failed almost in all survey situations. The operation errors are: $2.9 \pm 0.4 \%$ for $V_{v}$ and $3.6 \pm 0.1 \%$ for $V_{h}$ for VTI model. The velocity anisotropies, $\left(V_{v}-V_{h}\right) / V_{v}$, are: $1.7 \pm 0.4 \%$ along the layering, $14.2 \pm 1.8 \%$ along the diagonal, and $24.7 \pm 0.5 \%$ along the symmetry axis. The intrinsic velocity anisotropy of the modeling material is $25.8 \%$.

## 6. CONCLUSIONS

Releasing the assumptions that were made in deriving the formula for processing velocity analysis by hyperbola fitting, a new formula called skew hyperbolic moveout formula was proposed twenty years ago. The skew formula in form is a function of traveling distance and three velocity parameters: $V_{v}, V_{\gamma}$, and $V_{h}$. Among all of the three velocity parameters, $V_{v}$ is commonly adopted to perform a time-to-depth conversion; both $V_{\gamma}$ and $V_{h}$ are sensitivity to the local anisotropy of the media, thus can be used to indicate the anisotropy of the media, though, $V_{h}$ exhibits more stable variation with the lithology.

For a transverse isotropy of periodic interleaving with thin isotropic layers and constant velocity ratio, the $V_{\gamma}$ would become $V_{v}$. Laboratory works showed that the skew formula

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could be rewritten. The new skew formula is described by traveling distance, $V_{v}$, and $V_{h}$, and is called simplified nonhyperbolic moveout formula. In the simplified formula, the primary benefit of estimating reflection depth using $V_{v}$ was physically conformed. Laboratory data shows that the velocity anisotropy computed from $V_{\nu}$ and $V_{h}$ is sensitive to the anisotropy of the media and can be used to indicate the lithology of the subsurface strata. Experimental results also showed that for a single stratum, regardless to isotropic or aanisotropic, the trajectory of reflections could be satisfactorily described by the simplified nonhyperbolic moveout formula.

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