A Method of Estimating the Block-Like Body Under the Ground from the Polarization Ellipse of the Magnetic Field

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ABSTRACT

The feasibility of using the broad-band Magnetotelluric (MT) method for locating a two-dimensional (2-D) block-like body is studied by means of numerical models. An example of its applicability for locating a block-like anomalous body indicates that the sign reversal frequency of the tilt angle, or the ellipticity responses of magnetic polarization relates to the depth of the center of the anomalous body, and also to the resistivity contrast of the anomalous body to the host rock, based on the relationship of the few nomograms are developed, that provide a new approach to the interpretation of the magnetic polarization data.

1. INTRODUCTION

The application of numerical techniques to magnetotelluric modelling has been developed in recent years. The majority of these techniques are based on the finite element method (Rijo, 1977; Kaikkonen, 1977) and the finite difference method (Jones and Pascoe, 1971; Brewitt-Taylor and Weaver, 1976). In this work the finite element technique is used to compute the magnetotelluric responses over a two-dimensional block-like body. The element matrix equations are derived from the minimization of the electromagnetic energy (Coggon, 1971). The boundary values used in the global matrix equation are the same as those used by Pascoe and Jones (1972). The global matrix equation shown in the algorithm is solved using Gaussian elimination.

Using this algorithm, a series of theoretical MT response distributions have been computed for a variety of resistivity contrasts and burial depths. These responses of apparent resistivity or phase versus frequency are presented for both to TE (Transverse Electric) mode and the TM (Transverse Magnetic) mode. Finally the tilt angle and ellipticity of the magnetic field polarization (Smith and Ward, 1974) versus the frequency are evaluated.

From these pseudosections, it appears that the tilt angle and ellipticity will give more information about buried block-like anomalous bodies than the conventional apparent resistivity and phase pseudosections will starting from the pseudosections for the tilt angle and ellipticity versus frequency, we construct the nomograms for interpretation of the magnetic polarization data. The principal steps in this procedure are: (1) Preparing different pseudosections for the tilt angle and ellipticity versus frequency by (a) Varying the depth of the center of the anomalous body and computing the MT response, and (b) Varying the resistivity contrast between the anomalous body and the host rock and computing the MT response; (2) Constructing nomograms based on the relationship between the sign reversal frequency versus resistivity contrast and the depth of the buried body. A quantitative interpretation of the magnetic polarization response for these anomalous bodies using the nomograms will also be given in this study.

2. MODEL RESULTS

In this section, the results of modelling for two- dimensional block-like anomalous bodies are presented.

a. Effects of the overburden

Fig. 1 illustrates the pseudosections of the apparent resistivity and phase versus frequencies for a case with a two-dimensional block-like body of a 3×4 unit area (2 *km* per unit). In this model the depth of the buried body and the resistivity contrast between the buried body and the host rock, have been assigned the values of 2 units and 20 *ohm-m*/100 *ohm-m* respectively. Fig. 1a is the geoelectric model, Fig. 1b is the apparent resistivity plots for the TE and TM modes and Fig. 1c is the phase anomalies for the TE and TM modes. From these pseudosections, anomalies are presented as conductivity inhomogeneity exists.

Fig. 2 shows the MT responses of a horizontal layer over the model shown in Fig. 1. The difference between the model shown in Figs. 2a and 2b is that the former model has a resistive overburden while the latter has a conductive overburden.

From the comparison of the results shown in Figs. 1 and 2, it indicates that there is no obvious characteristic to be used to identify these type of underground structure, especially for the conductive overburden.

Fig. 3 shows the same geoelectric model as shown in Fig. 1a, it is obvious

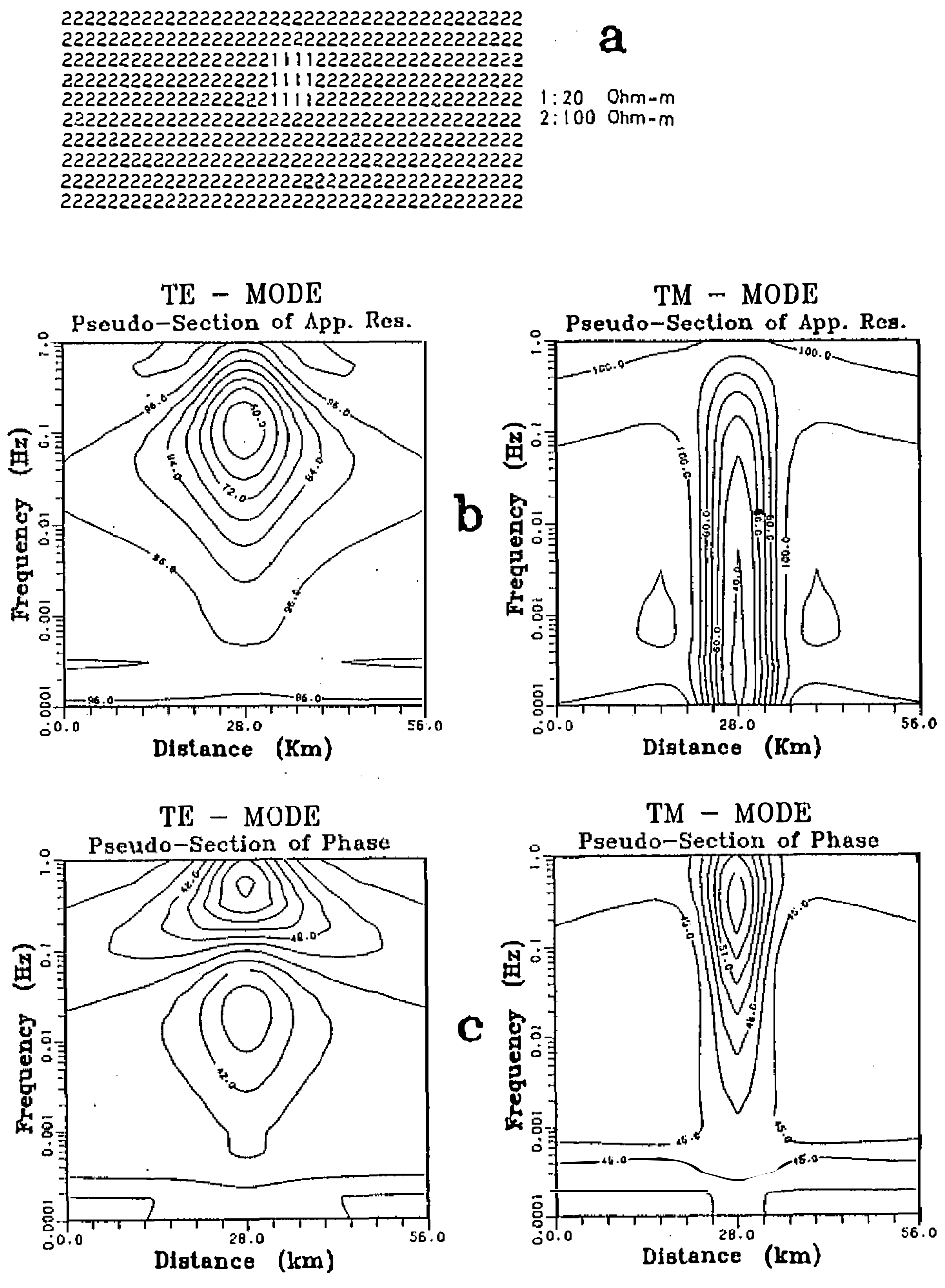

Fig. 1. (a) The geoelectric model. (b) The pseudosections of apparent resistivity anomalies for the TE and TM modes. (c) The pseudosections of phase anomalies for the TE and TM modes.

that the characteristics of sign reversal of the tilt angle and ellipticity anomaly for the TE mode can be used to identify the existence of the "block-like" body. Fig. 4 has the same type of geoelectric models as Fig. 2. As pointed out by Rijo

(1977), the presence of flatly-layered overburden has little effect on the value of the sign reversal frequency, the frequency of the nodal point (circle mark as shown in Fig. 4), the ellipticity or tilt angle anomalies.

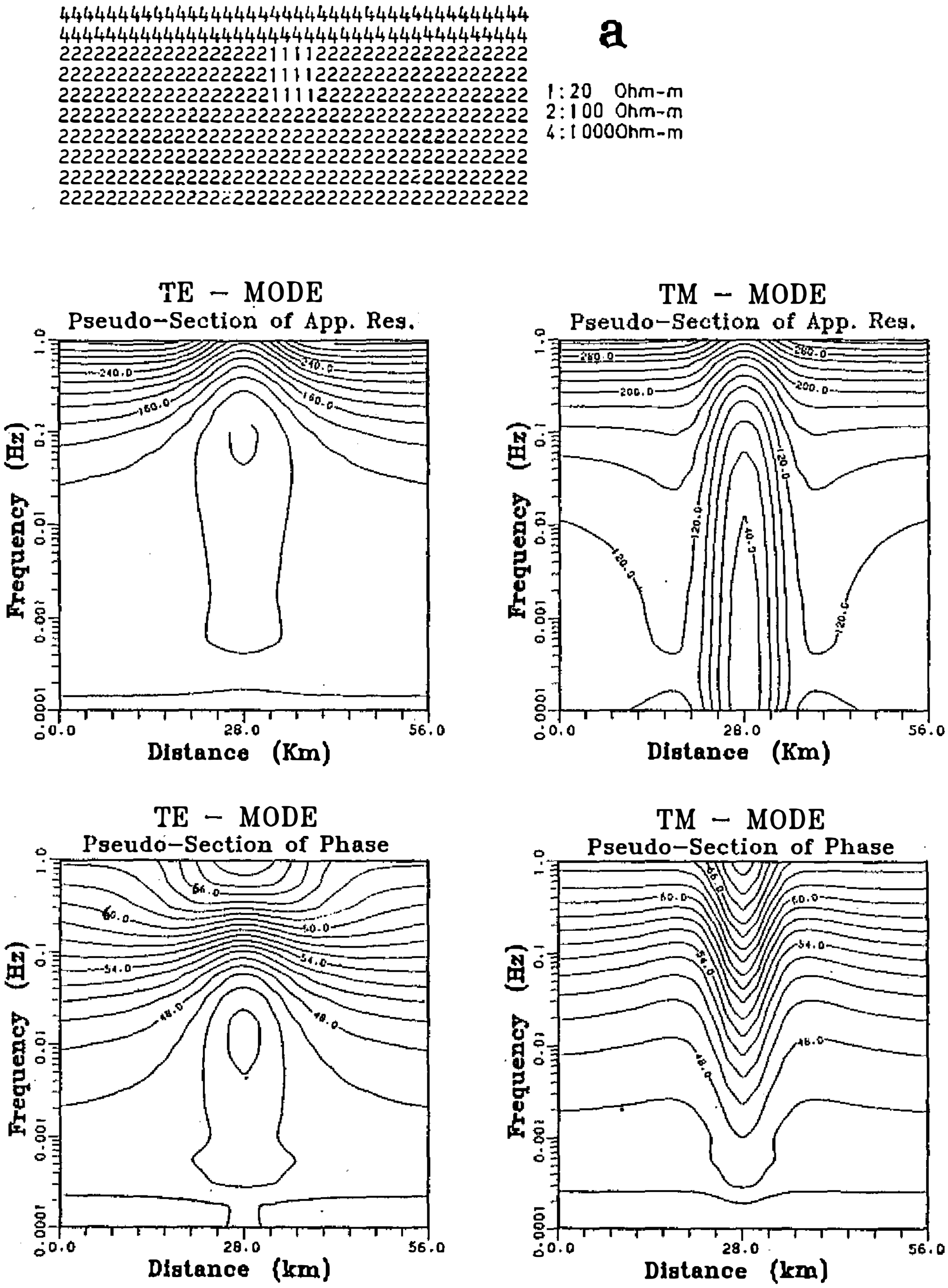

Fig. 2. (a) Effect of a highly resistive flat layer over the model shown in *Fig.* 1a. (b) Effect of a highly conductive flat layer over the model shown in *Fig.* 1a.

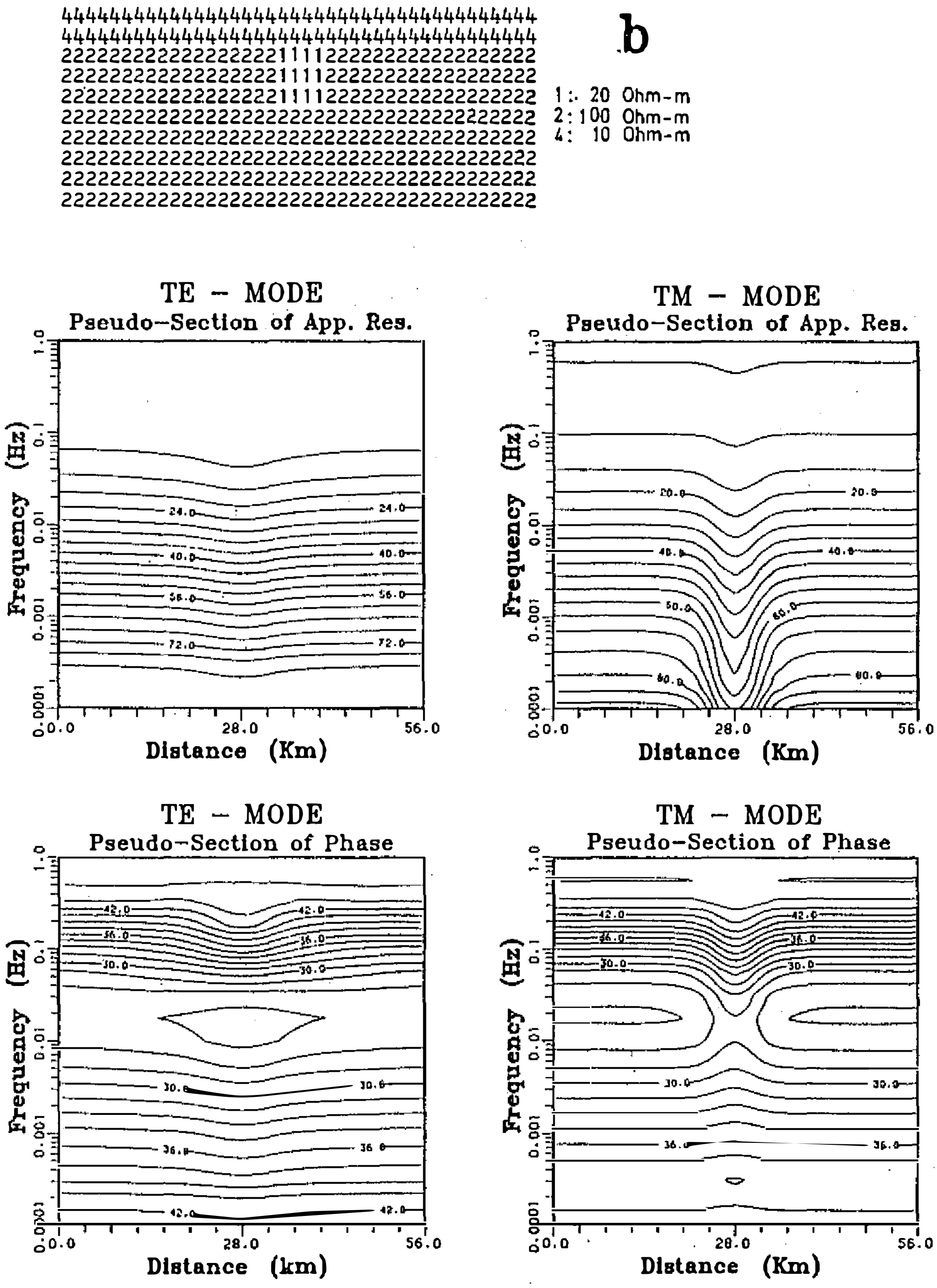

Fig. 2. (Continued)

b. Effect of depth

From the comparison of the reversal frequency of the ellipticity and/or tilt angle responses shown in Figs. 3 and 5, the increase in depth of the center of

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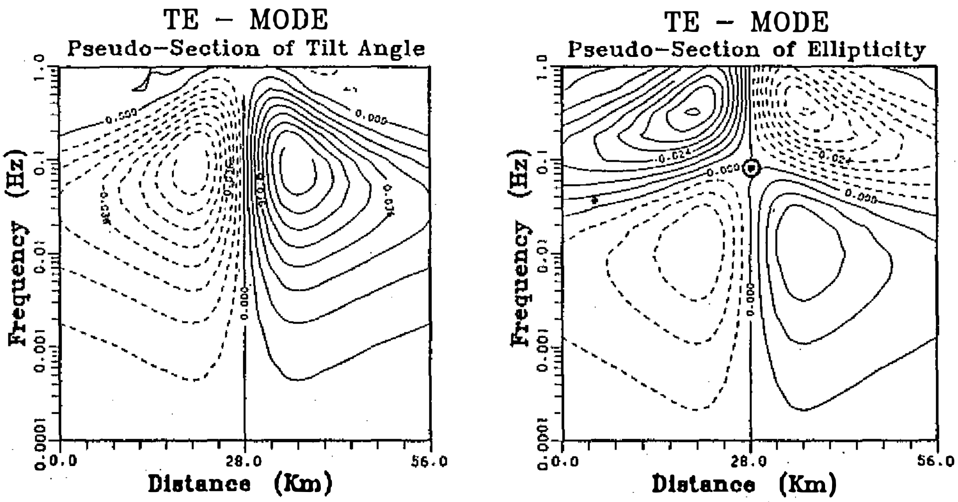

Fig. 3. Pseudosections of tilt angle and ellipticity anomalies for the TE mode for the same geoelectric model as shown in *Fig.* 1a, it shows that the ellipticity (or tilt angle) has the sign reversal characteristic.

an anomalous body will result in the decrease of the sign reversal frequency of ellipticity (or tilt angle). The linear frequency (in log space) dependence of the depth of the anomalous body can be successfully used to locate the underlying body center provided that the resistivity contrast is known in advance for no overburden case.

c. Effect of the size

The results shown in Figs. 3 and 6 indicate that the sign reversal frequency of ellipticity (or tilt angle) depends only on the center of the anomalous body, and is independent of the size of the "block-like" body.

d. Effect of resistivity contrast

As shown in Figs. 3 and 7, when the resistivity contrast of the anomaly to the host rock is a constant, the frequency at which the ellipticity (or tilt angle) reverses its sign depends on the skin depth of the host rock.

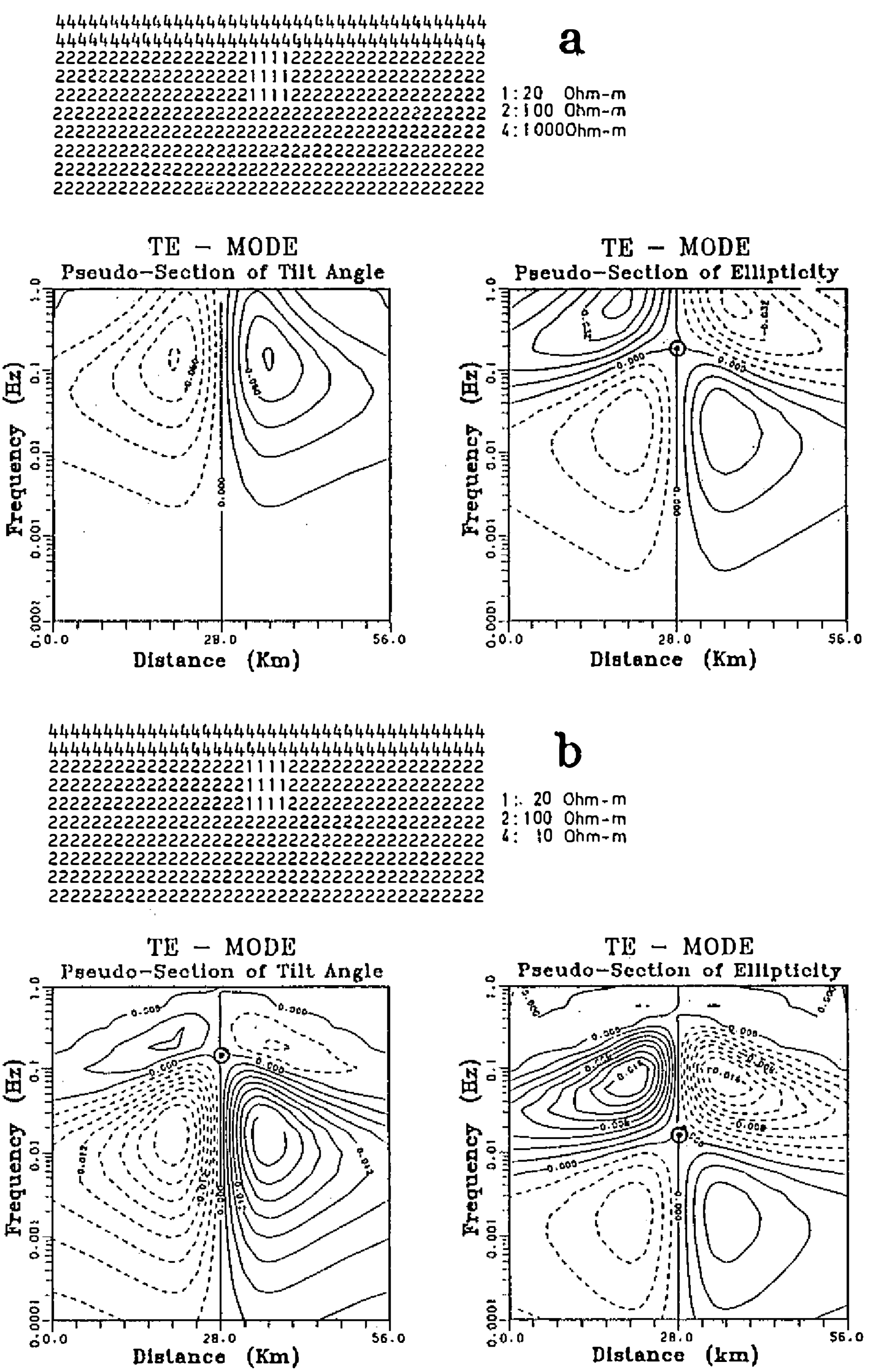

Fig. 4. Pseudosections of the tilt angle and ellipticity anomalies in the same models as shown in *Fig.* 2. (a) A highly resistive overburden presented. (b) A highly conductive overburden presented. It shows that the presence of a flatly-layered overburden does not affect the characteristic of sign reversal.

3. NOMOGRAMS

As noted therein, the response attributable to a block-like anomalous body buried in a half space could be discriminated when the tilt angle and ellipticity

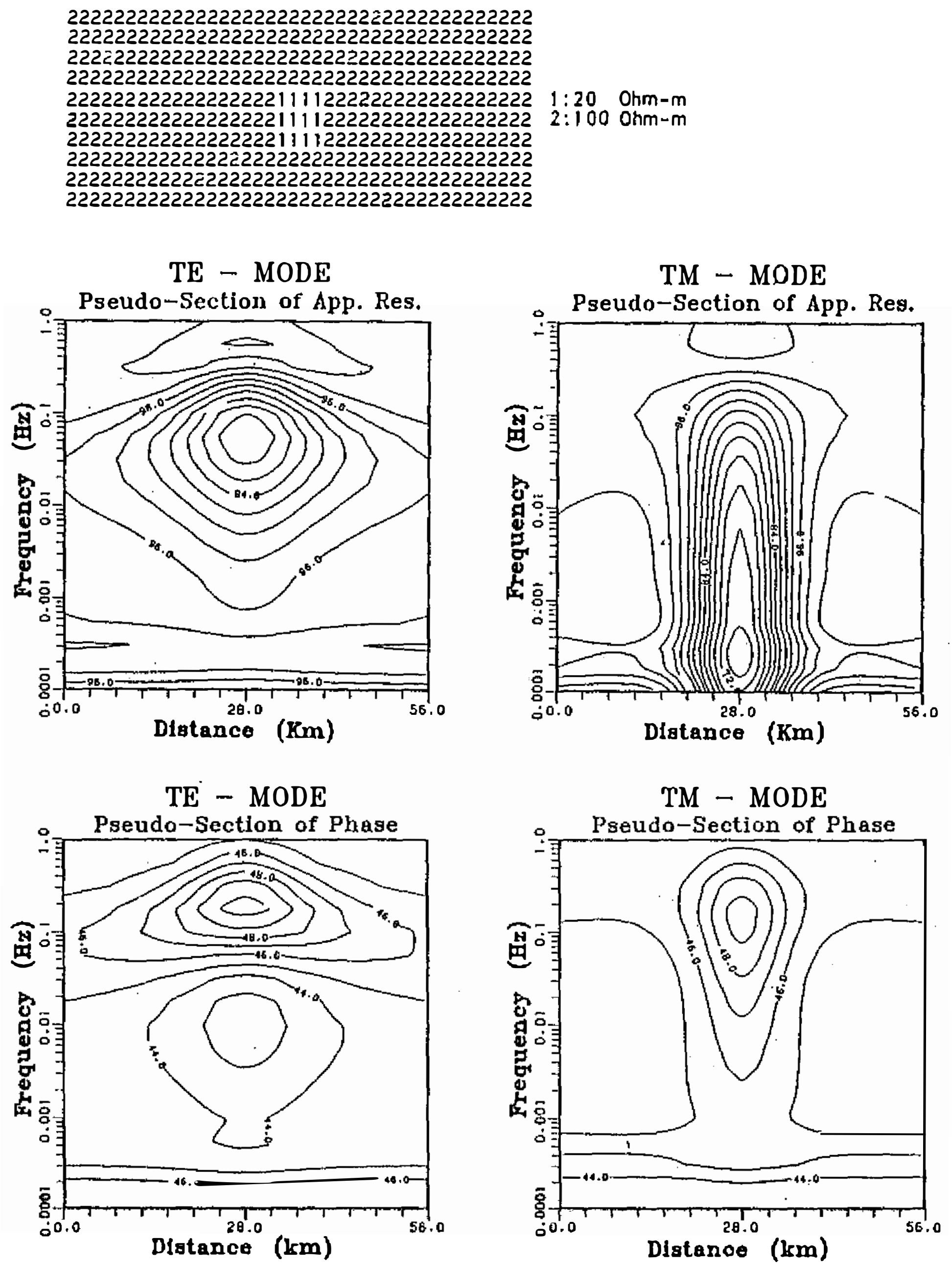

Fig. 5. Pseudosections related to the depth change (2 units to 4 units) in the model as shown in *Fig.* 1.

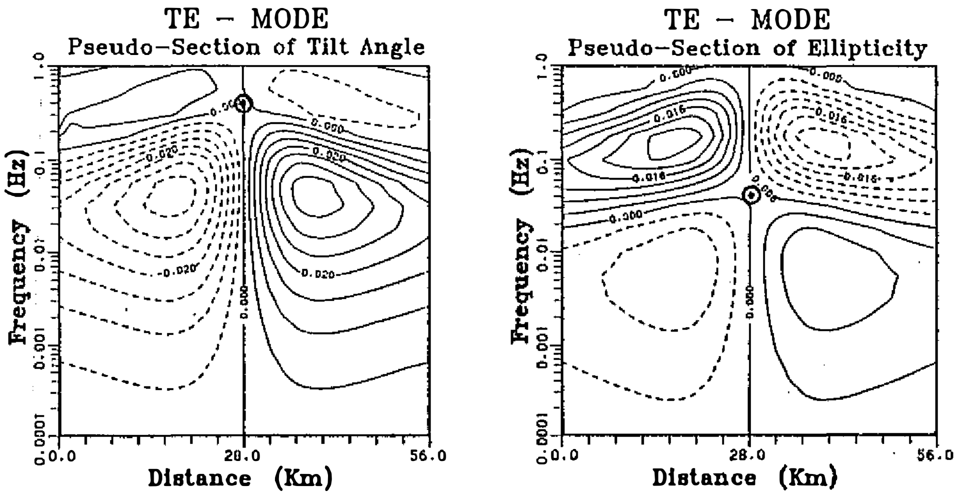

Fig. 5. (Continued)

values were contoured in frequency-distance space. Fig. 8a shows the relation-

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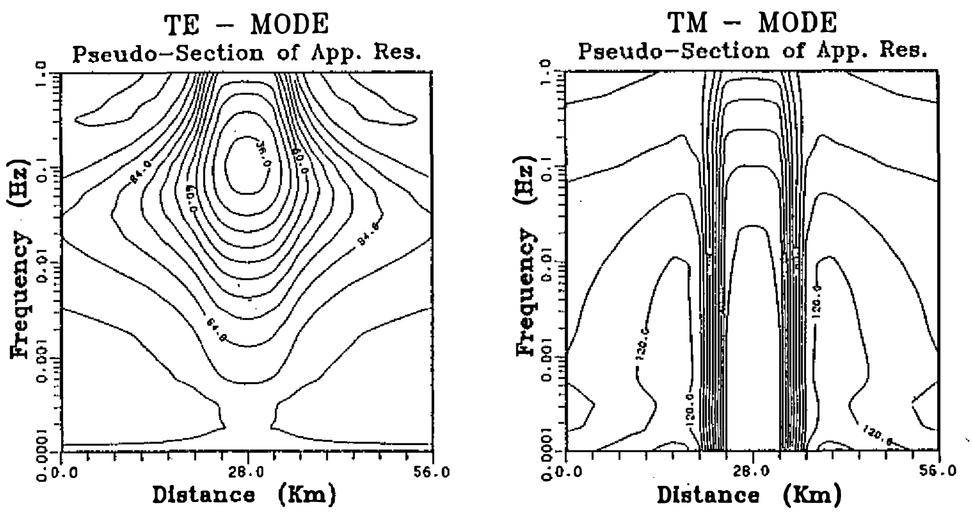

Fig. 6. Pseudosections related to the size change (3x4 unit area to 5x6 unit area) in the model shown in *Fig.* 1.

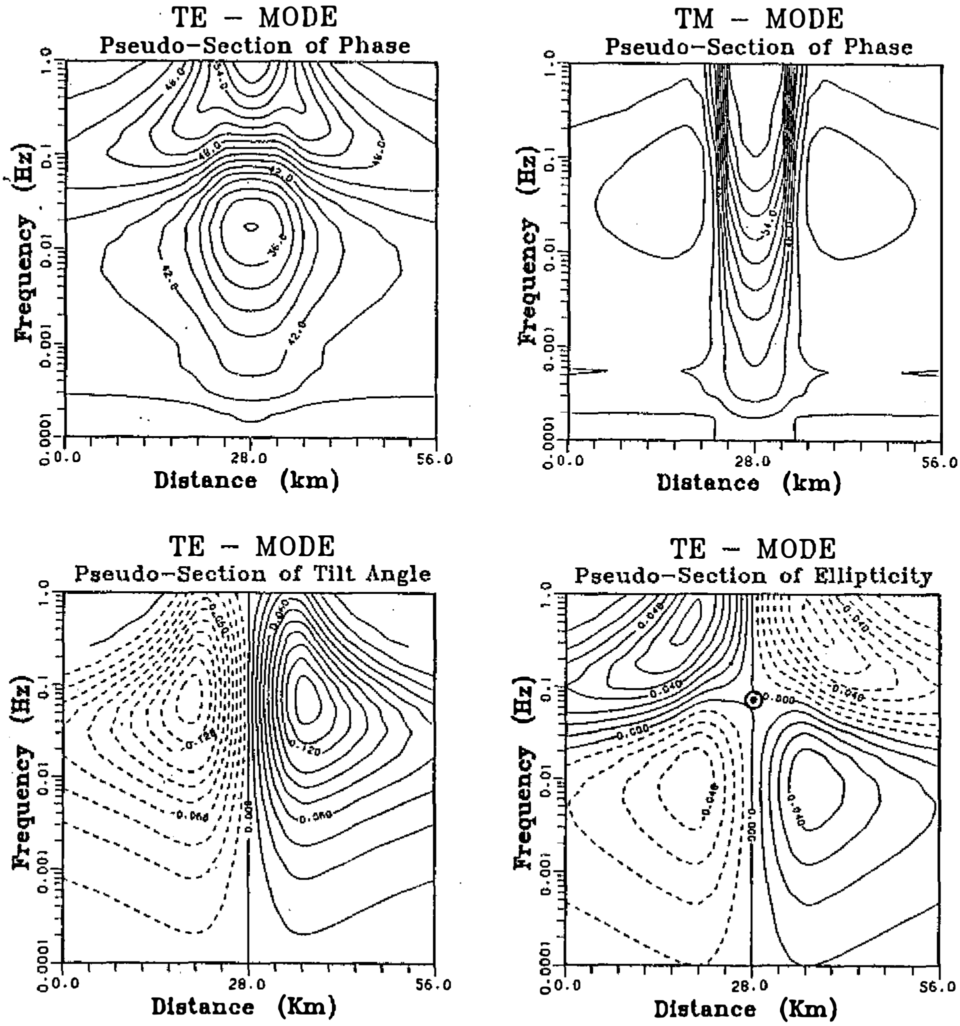

Fig. 6. (Continued)

ship between the depth of the center of a buried body and the sign reversal frequency of the ellipticity for different resistivity contrasts. Fig. 8b shows the relationship between the sign reversal frequency of the tilt angle and the ellipticity for different resistivity contrasts and the depth of the center of the buried body. In order to use the model results to interpret MT data, we construct the nomograms as shown in Fig. 9. These two sheets would help us to take quantitative interpretation of broad-band magnetotelluric data for a block-like buried body.

If the sign reversal frequency of ellipticity and tilt angle from the MT responses over an expected anomaly of block-like body are known then we may use the nomograms to estimate the resistivity contrast and determine the depth of the buried body. An example of using these nomograms is given in the next section.

4. STEPS FOR INTERPRETATION OF THE MT DATA

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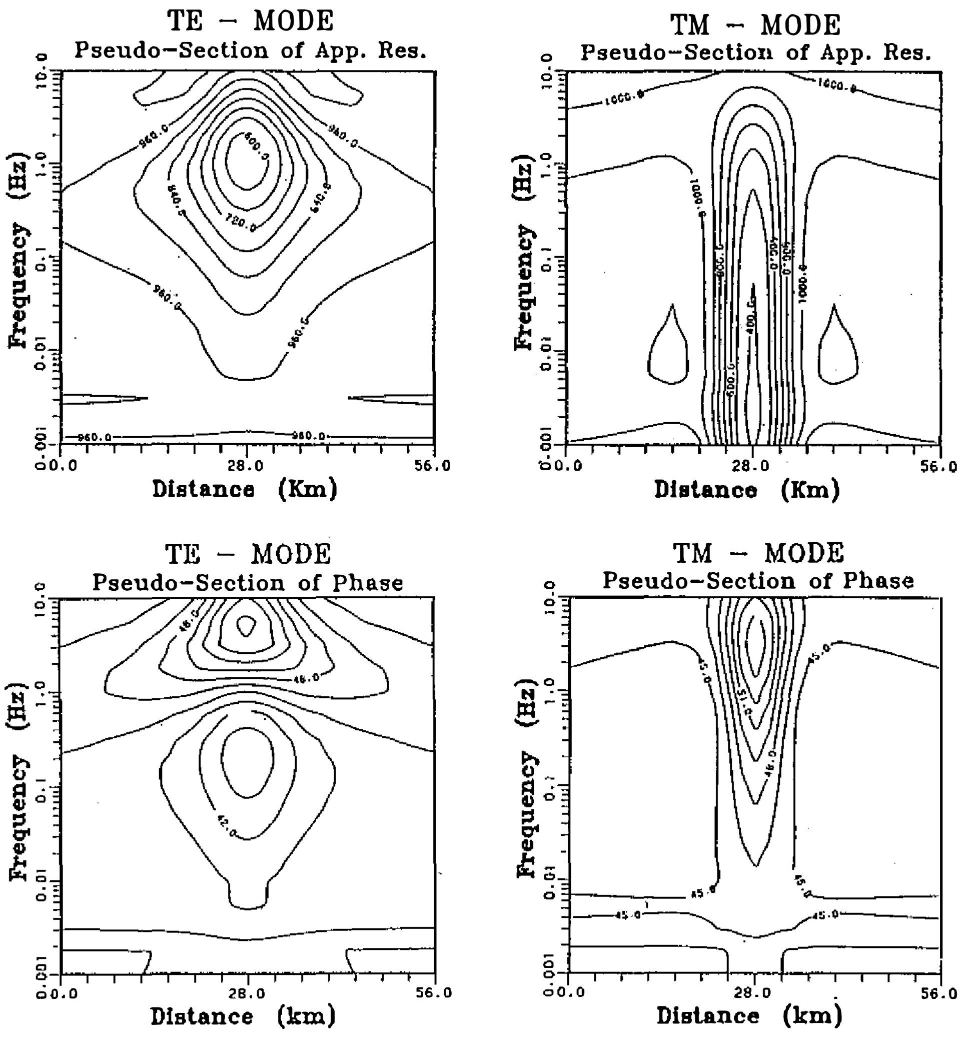


Fig. 7. Pseudosections related to the model with constant resistivity contrast (1:5) as shown in *Fig.* 1. The sign reversal frequency of the tilt angle (or ellipticity) depends on the skin depth of the host rock.

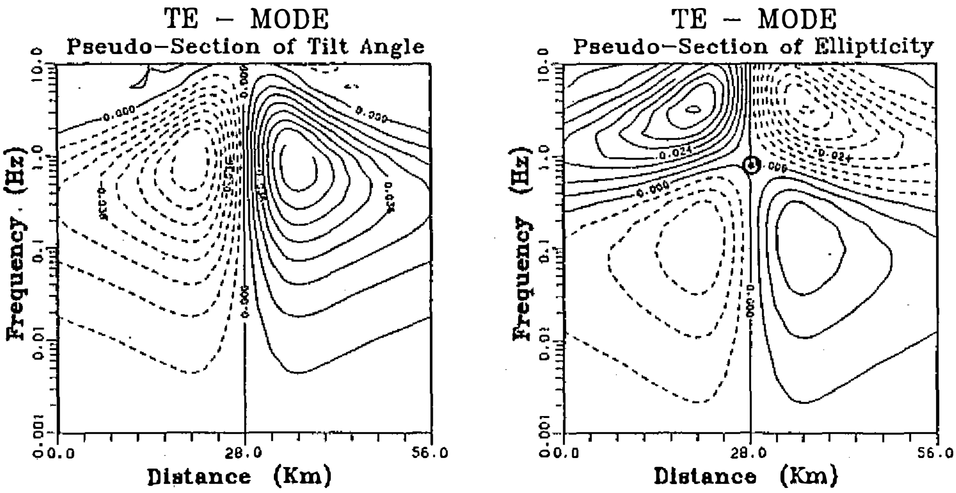


Fig. 7. (Continued)

An example of a step-by-step interpretation procedure will be given in this section for the MT data by means of the newly-developed nomograms. The resistivity of the host the rock does affect the sign reversal frequency. First, we estimate the resistivity of the host rock from the apparent resistivity profile. In our example, the resistivity of host rock is 200 ohm-m.

If we have the sign reversal frequency from the ellipticity pseudosection only, then nomogram sheet 1 is available, and the following steps are suggested.

(1) In this example, the sign reversal frequency estimate for the ellipticity is 0.06 Hz. On the transparent bilogarithmic paper, draw a horizontal line through a point in the vertical coordinate of the sign reversal frequency (0.06 Hz), and set the origin to be at 0.01 Hz, as shown in Fig. 10a.

(2) Superimpose the transparent paper on Fig. 9a and move one sheet with respect to the other, keeping the vertical axes parallel with each other until the origin of the transparent paper is coincident with a point 200, the value of the resistivity of the host rock on the vertical coordinate of nomogram sheet 1, as shown in Fig. 11a. The horizontal line will cross a set of possible resistivity contrast curves related to different depths. If the information of the resistivity contrast is available, then the depth of the anomalous buried body can be estimated.

If we could find the sign reversal frequencies for both tilt angle and ellipticity from the observed pseudosections, then nomogram sheet 2 is available, the following steps are a generalized approach for this study.

(1) Plot the pseudosections of ellipticity and tilt angle versus frequency separately from the data collected during a reconnaissance survey. Mark the sign reversal frequency observed on the pseudosections. These fre-

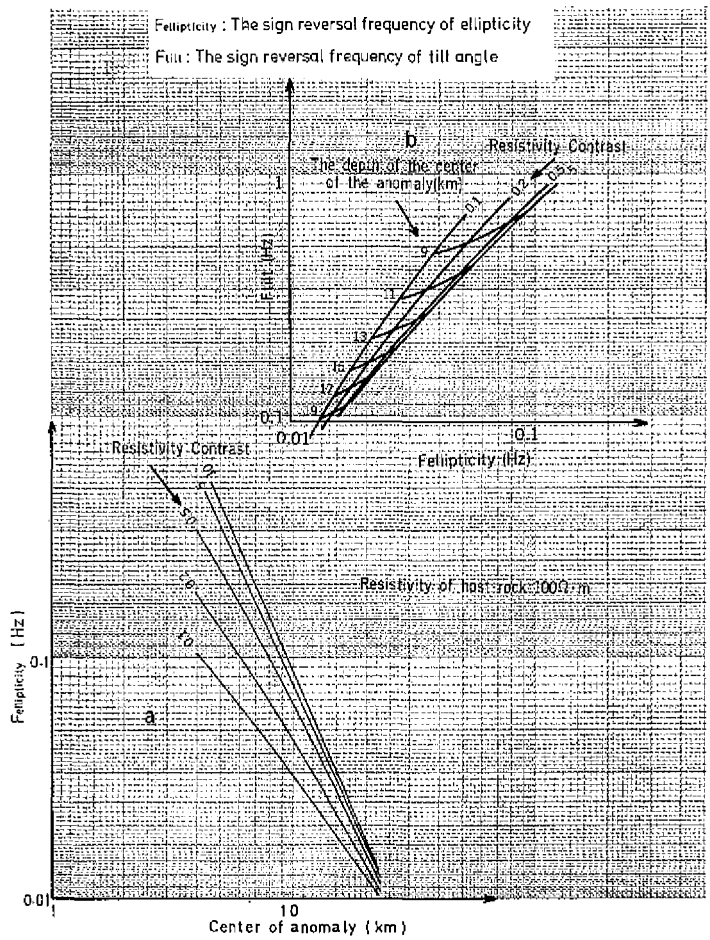
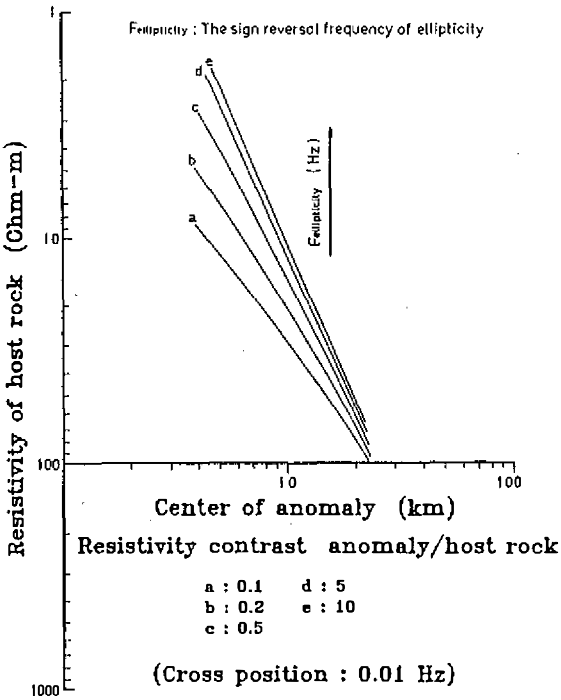


Fig. 8. The modelling results. (a) The relationship between the depth of the center of the anomaly and the sign reversal frequency of the ellipticity with different resistivity contrasts. (b) The relationship between the sign reversal frequency of the tilt angle and ellipticity for different resistivity contrasts and the depth of the center of the anomalous body.

quencies do relate to the geological structure. In this example the sign reversal frequency estimated for tilt angle and ellipticity are 0.6 Hz and 0.06 Hz respectively. Mark the coordinates of point A with a known value for the sign reversal frequency of the tilt angle (0.6 Hz, i.e. horizontal coordinate) and the sign reversal frequency of the ellipticity (0.06 Hz, i.e. vertical coordinate) on a transparent bilogarithmic paper, as shown in Fig. 10b. The origin of this log-log-plots will be at the point (0.01 Hz, 0.1 Hz).

(2) Place the origin for the the log-log-plots at the point (coordinate 200,200), a value of host rock, on nomogram sheet 2, as shown in Fig. 11b indica-

NOMOGRAM Sheet 1



NOMOGRAM Sheet 2

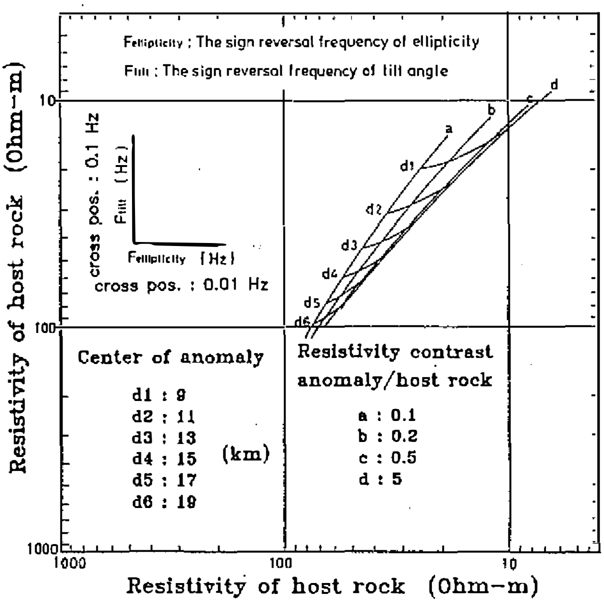


Fig. 9. (a) Nomogram sheet 1. (b) Nomogram sheet 2.

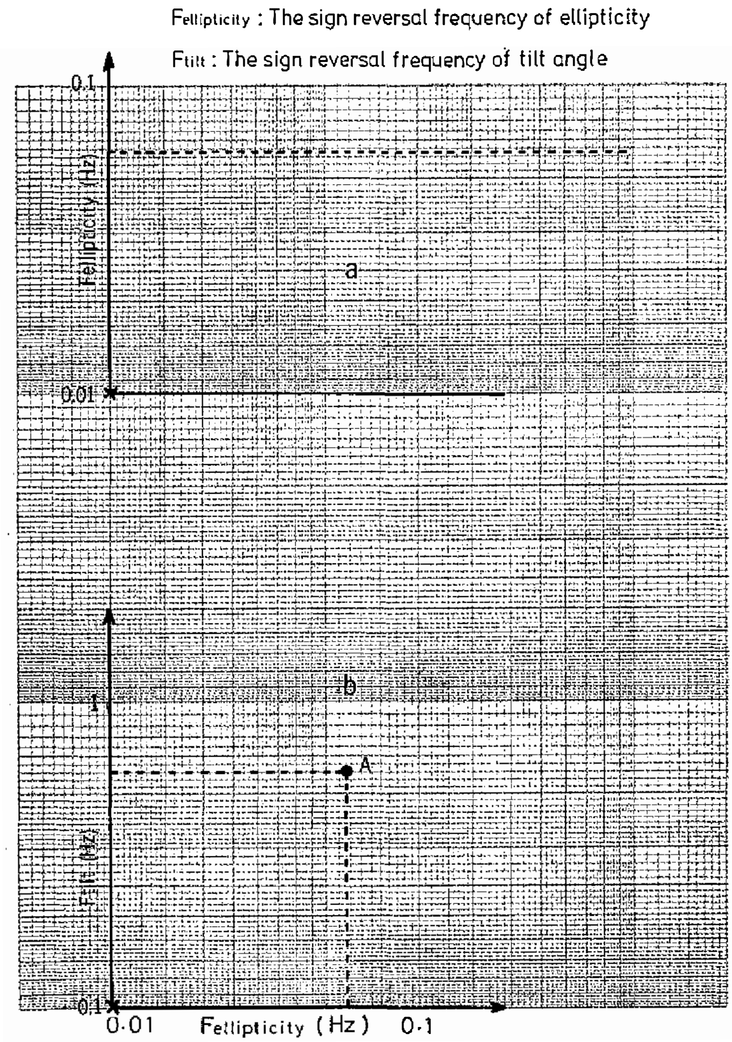


Fig. 10. Example for illustrating the use of nomograms. (a) When sign reversal frequency from the ellipticity pseudosection is available only. (b) When both sign reversal frequencies for both tilt angle and ellipticity are available.

tion a set of master curves which show the effects of the depth of center of the anomaly and the resistivity contrast for a block-like body buried in a half space. Keep axes parallel, read the coordinates corresponding to the position of point A on the logarithmic paper and then the curve will give the depth of center for the anomalous body and the resistivity contrast curves will give the resistivity contrast of the anomaly to the host rock. Interpolation between master curves is permitted. The resistivity of the anomalous body can be figured out by the known value of the resistivity of the host rock.

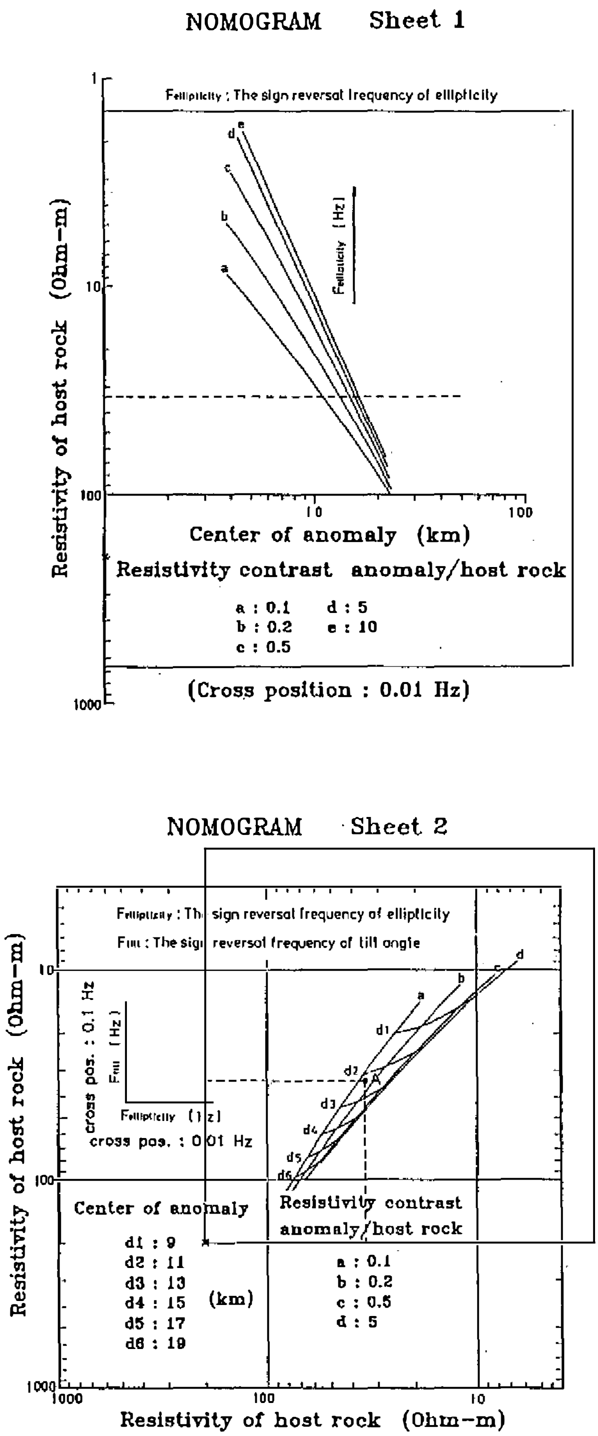


Fig. 11. (a) The interpretation procedure when the sign reversal frequency of the ellipticity is available only. (b) The interpretation procedure when the sign reversal frequencies of the tilt angle and ellipticity are known.

5. CONCLUSION

The sign reversal frequency of the ellipticity anomaly and tilt angle anomaly are related to the depth of the body and resistivity contrast between an anomalous body and the host rock. This characteristic is not affected much by the presence of the overburden.

Furthermore we have presented a method for the rapid evaluation of this type of MT anomalies. The newly-developed nomograms give much information about the anomalous block-like body buried in a half space. This quick and reasonably accurate method may help in the analysis of the broad-band magnetotelluric data when reliable estimates of the requisite MT functions are given.

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應用磁場極化橢圓來推估地下塊狀體法

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應用數值法模擬寬頻磁流電磁法對地下二維塊狀構造體之響應，依磁場反應之傾角或橢圓率與頻率關係圖，顯示傾角或橢圓率更換點之頻率與塊狀體中心之深度及塊狀體與圍岩之電阻率比值相關。據此特性，可作各相關參數間之列線圖，提供應用磁場極化資料來推估地下塊狀體特性之一可行性方法。