TAO, Vol.6, No.2, 181-195, June 1995

A Review of Receiver Function Techniques for Estimation of One-Dimensional Velocity Structures

CHENG-HORNG LIN[1]

(Manuscript received 21 August 1994, in final form 21 November 1994)

ABSTRACT

Two receiver function techniques both broadly used by seismologists to estimate one-dimensional shear wave velocity structures of the crust and upper mantle beneath seismic stations have been evaluated. One employs a deconvolution filtering, which is directly accomplished in the time domain (Vinnik, 1977; Kosarev *et al.*, 1987). The other is completed through a source-equalization, which is performed by spectra division in the frequency domain (Langston, 1977; Owens *et al.*, 1984). In this study, the performance of these techniques is examined from two sets of synthetic seismograms that are computed from two one-dimensional models by using Thomson and Haskell's method (Haskell, 1962). The results suggest that both techniques can mostly recover the assumed models very well when the energy of multiples (or crustal reverberations) in the direction of P-wave propagation is minor. If the model is more complicated or the multiple energy becomes stronger, however, the results from Vinnik-Kosarev technique appear to be better than the other in both modeling the converted P-SV phases and inverting the structures. Combining the results of the synthetic tests and theoretic comparisons, it is concluded that the differences resulting from both techniques are primarily caused by the processes through different domains. Besides, both techniques consistently indicate that the inversion results are dependent upon the incident angles. For the nearly vertical incidence, neither technique could resolve the models very well.

(Key words: Receiver functions, 1-D structures)

1. INTRODUCTION

The modeling of converted phases of P-SV type from teleseismic waveforms, known as receiver function analysis, has often been used to determine one-dimensional shear wave velocity structures of the crust and upper mantle beneath a seismic station. One main process

[1] Institute of Earth Sciences, Academia Sinica, P. O. Box 1-55, Nankang, Taipei, Taiwan, R.O.C.

of these analyses is to recover the converted phases by removing the source and deep mantle effects from tele-seismograms. These converted phases are generated at the boundaries between homogeneous layers of the crust and upper mantle which the primary waves pass through. To carry out this process, two different kinds of techniques have been independently developed. One is the source-equalization that was first proposed by Langston (1977) to deal with long-period teleseismic P-waves collected from several World-Wide Standard Seismograph Network (WWSSN) stations. Then Owens *et al.* (1984) and Owens (1987) incorporated this technique with an inversion scheme in the time domain to recover detailed information present in the broadband teleseismic P-waveforms recorded at several stations of the Department of Energy Regional Seismic Test Network (RSTN). The other technique, initially reported by Vinnik (1977) and later modified by Kosarev *et al.* (1987; 1993), is the procedure of using the time domain deconvolution filtering of Berkhout (1977).

This study is undertaken in order to evaluate the two techniques mentioned above, and it is based on synthetic tests and theoretic comparisons. At the beginning of this study, both technical procedures of obtaining receiver functions are briefly described. Then both techniques are examined by modeling synthetic seismograms that were created by the Thomson-Haskell method (Haskell, 1962). Finally, the results of the synthetic tests along with some theoretic comparisons are discussed.

2. METHODOLOGY

2.1 Langston and Owens Technique

Langston (1979) and Owens *et al.* (1984) used a source-equalization procedure to determine the velocity structure of the crust and upper mantle beneath a seismic station (Figure 1a). They stated that a teleseismic P-wave, $D(t)$, recorded at a station can be theoretically expressed by :

$$\begin{aligned} D_V(t) &= I(t) * S(t) * E_V(t), \\ D_R(t) &= I(t) * S(t) * E_R(t), \\ D_T(t) &= I(t) * S(t) * E_T(t), \end{aligned} \tag{1}$$

where the subscripts V, R, and T are the vertical, radial, and tangential components, respectively; $I(t)$ is the instrument response of the recording system; $S(t)$ contains the effects of the seismic source and deep mantle, and $E(t)$ is the effects of the crust and upper mantle structure. Asterisks denote the convolution operator.

The main purpose of the source-equalization process is to isolate the response of crust and upper mantle structures from the other factors that interact with it to form the observed seismograms recorded at teleseismic distances. The most important assumption made in this technique is that $E_V(t) = \delta(t)$, which means crustal multiples (reverberations) and converted phases on the vertical component of steeply incident P-waves are considered negligible. Thus, $D_V(t) = I(t) * S(t)$. Moreover, assuming that instrument responses are matched between components, the source and deep mantle effect in the radial component can be removed by deconvolving the vertical components, $D_V(t)$, from the radial component, $D_R(t)$.

As a matter of fact, the procedure for performing the deconvolution is to divide the spectrum of the radial component by that of the vertical component in the frequency-domain. The process is given by:

$$E_R(\omega) = \frac{D_R(\omega)}{D_V(\omega)} \tag{2}$$

To reduce the effects of noise generated by dividing by very small numbers due to troughs in the spectrum, a substitute water-level process (Dey-Sarkar and Wiggins, 1976) is given by:

$$E_R(\omega) = \frac{D_R(\omega) \cdot D'_V(\omega)}{F_{ss}(\omega)} \cdot G(\omega), \tag{3}$$

where

$$F_{ss}(\omega) = \max\{D_V(\omega)D'_V(\omega), c \max[D_V(\omega)D'_V(\omega)]\},$$

and

$$G(\omega) = \exp(-\frac{\omega^2}{a^2}).$$

$D'_V(\omega)$ is the complex conjugate of $D_V(\omega)$. The function of $F_{ss}(\omega)$ can be thought of as the auto-correlation of $D_V(\omega)$ with any spectral troughs filled to a level as determined by the water-level value "c". The parameter "a" is used to control the width of the Gaussian function. This substitute Equation (3) introduces the minimum allowable amplitude level of the amplitude spectrum of the vertical component.

After spectral division, the resulting spectrum is transformed back to the time domain to obtain the observed receiver function of the radial component, $E_R(t)$. Meanwhile, the calculated receiver function is generated by applying the source-equalization procedure described above to the synthetic seismograms that are calculated by a fast ray-tracing scheme (Langston, 1977). Finally, one-dimensional shear wave velocity structures are inverted by minimizing the difference between the observed receiver function and the calculated one.

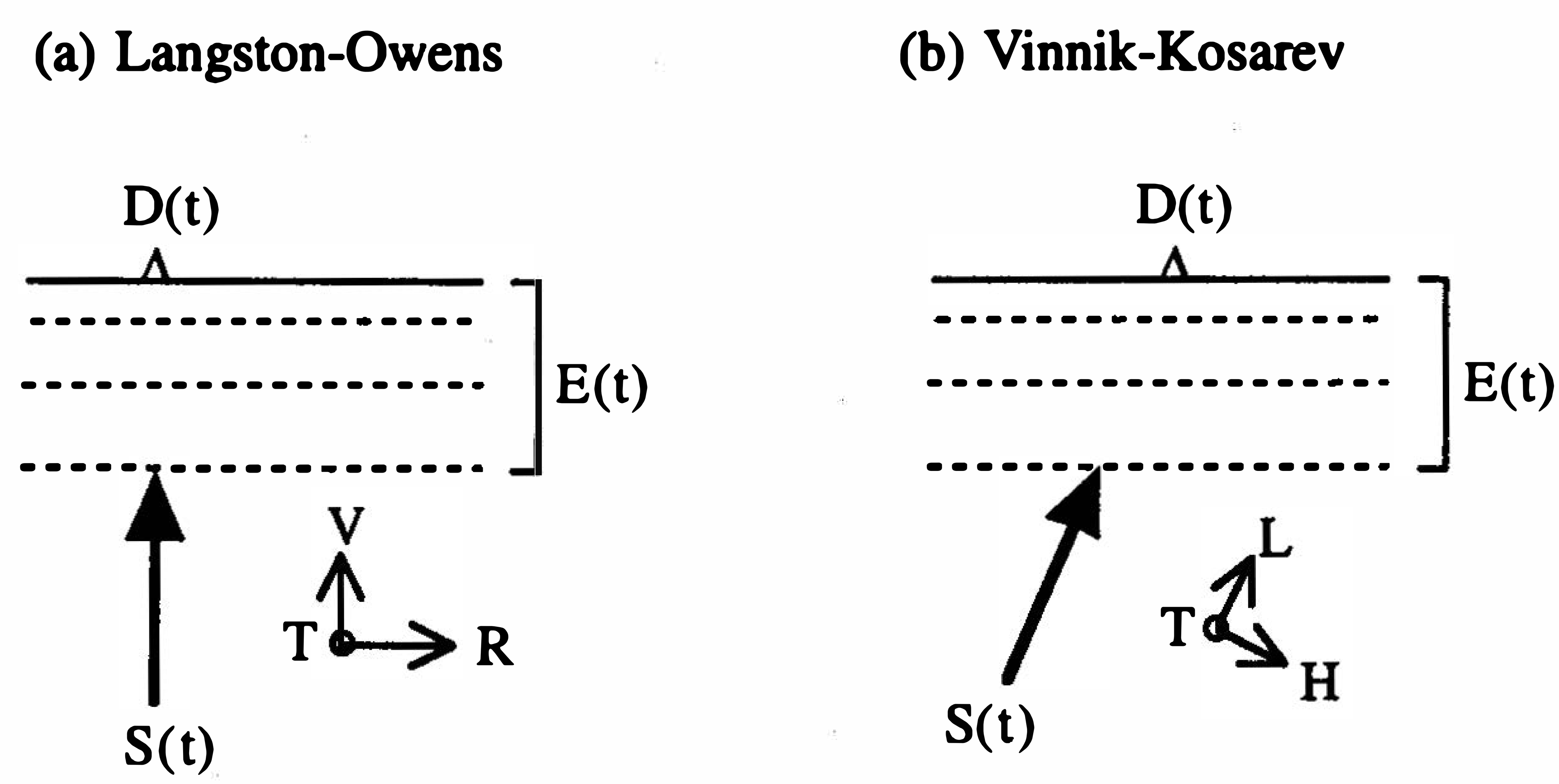


Fig. 1. Simplified models to indicate the crust and upper mantle structures and the incidence of teleseismic waves: (a) coordinate system (V, R, T) used in the Langston-Owens technique, and (b) coordinate system (L, H, T) used in the Vinnik-Kosarev technique.

2.2 Vinnik-Kosarev Technique

Vinnik (1977) and Kosarev *et al.* (1993) considered seismograms in an L, H, and T coordinate system, where L is along the P polarization direction, H is in the plane of propagation and normal to L, and T is normal to both H and L (Figure 1b). This coordinate system is chosen to optimize the detection of SV. These authors also assumed the instrument response to be the same in the three components. Thus, the response of the teleseismic P-wave, $D(t)$, can be presented as:

$$\begin{aligned} D_L(t) &= S(t) * E_L(t) \\ D_H(t) &= S(t) * E_H(t), \end{aligned} \tag{4}$$

where $S(t)$ is the source of the P-wave in the half-space, $E(t)$ is the response of the layer structure, and asterisks are the convolution operator.

Once the seismograms are rotated, three steps are followed. The first step is to generate a deconvolution filter, $F(t)$, in the time domain by minimizing the least-square difference between the output of the filtering operating on the L component and a spike-like function of the normalized amplitude. This deconvolution filter is essentially equivalent to the Wiener filter (Berkhout, 1977), and its purpose is to recover the contributions of the P-SV phase conversions to the H component. The second step is the application of this deconvolution filter to the H component, which results in the response of the medium in the H direction to a normalized spike in the L direction. This step is valid only if the L component is not contaminated significantly by multiples from near-surface structures, which means the structure response in the L component, $E_L(t)$, is considered as a spike-like function. The final step is to invert the structure by minimizing the difference between the observed receiver functions and the computed responses of the medium.

3. SYNTHETIC TESTS

To evaluate the performance of both techniques mentioned above, two groups of synthetic seismograms have been created using the Thomson-Haskell technique (Figure 2). The synthetic seismograms are constructed from two assumed one-dimensional models (Table 1). The M1 model is composed of three layers with a half space, and velocities gradually increase with depths. In contrast, the more complicated M2 model is composed of seven layers with a half space, including a low velocity layer at depths between 6 and 10 km. These synthetic seismograms are the structure responses to a spike-like function of P input with a sampling interval of 0.2 seconds. The width of the spike is 2 seconds. Each group includes four pairs of synthetic seismograms corresponding to four representative incidences (I=1°, 10°, 20° and 30°), respectively.

The synthetic seismograms clearly show that the energy of multiples (or reverberations) and converted P-S phases is strongly dependent upon the incident angles of the input seismic waves as well as the models (Figure 2). The energy of the converted P-SV phases significantly increases with the incident angles. For the 30° incidence, the maximum amplitude of the P-S converted phases reaches about 10% of the first P-wave. For the nearly vertical incidence (I=1°), on the other hand, there is almost no energy of the P-SV converted phases in the H-component. Moreover, the energy of multiples decreases as the incident angle becomes

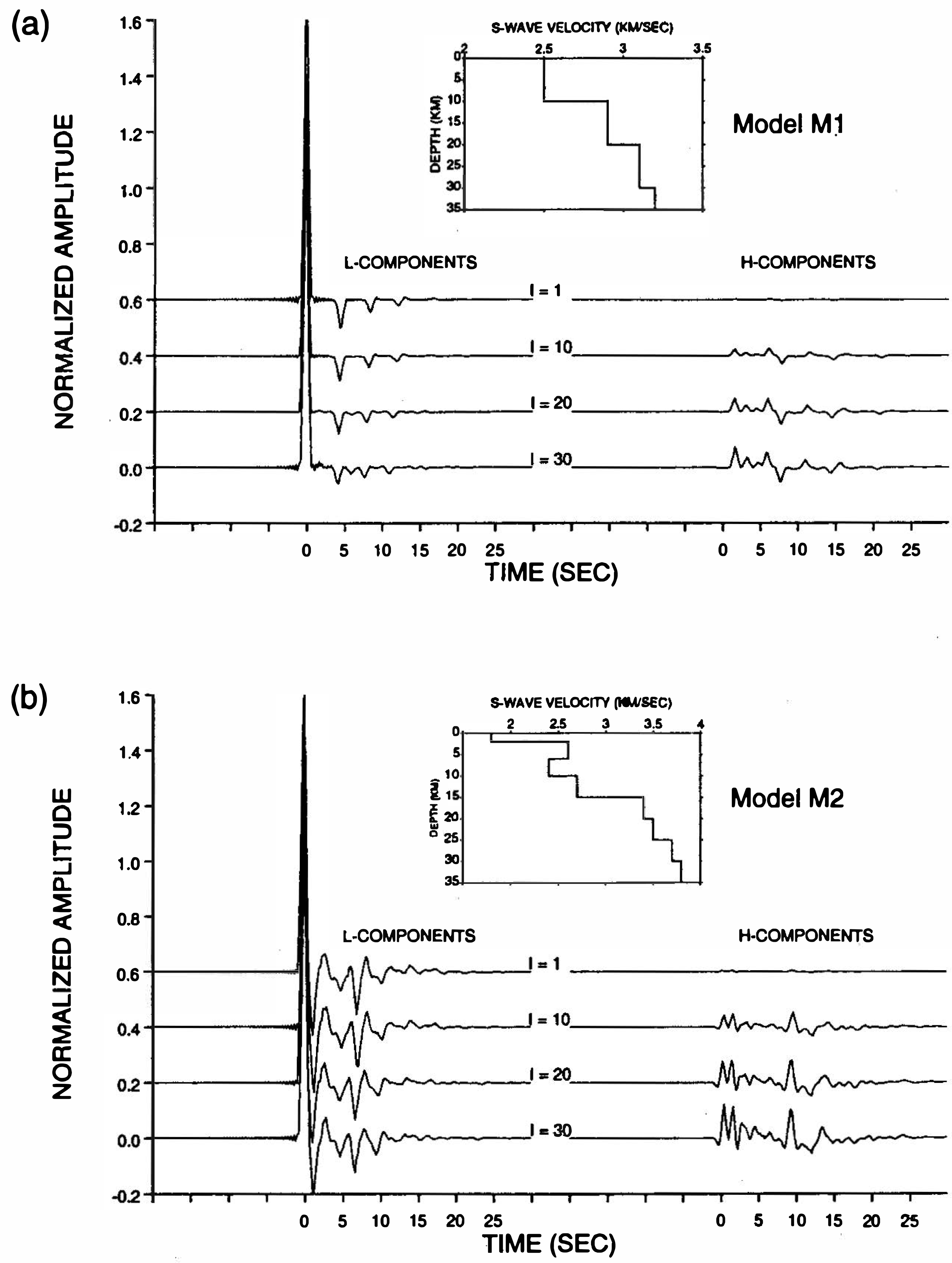


Fig. 2. Synthetic seismograms computed at four different incident angles (1°, 10°, 20° and 30°): (a) constructed from the M1 model, and (b) constructed from the M2 model.

horizontal. To illustrate, the energy of multiples in the L components from 1° incidence is about twice that from 30° incidence for the case of model M1.

Figure 2 also shows that the multiples and converted P-SV phases computed from the two models are quite different in not only shape but also amplitude. For example, the average energy as shown in Figure 2a is only about half of that in Figure 2b. Besides, it should be noted that the maximum energy of the multiples in the L-component is about 10-15% of the first P-wave arrivals, depending on the model and incidence angle. These results are not surprising because the reflected or transmitted energy from each boundary of the model is determined by the velocity and density contrast between the layers.

Table 1. Two one-dimensional models for generating synthetic seismograms.

M1				M2			
Vp (km/s)	Vs (km/s)	Density (g/cm^3)	Depth (km)	Vp (km/s)	Vs (km/s)	Density (g/cm^3)	Depth (km)
4.38	2.5	2.46	0	3.15	1.8	2.26	0
				4.55	2.6	2.49	2
				4.20	2.4	2.44	6
5.07	2.9	2.58	10	4.72	2.7	2.52	10
				5.95	3.4	2.73	15
5.43	3.1	2.64	20	6.12	3.5	2.76	20
				6.47	3.7	2.88	25
5.60	3.2	2.67	30	6.65	3.8	2.94	30

3.1 Tests of the Langston-Owens Technique

To test both techniques, the synthetic responses as shown in Figure 2 are taken to be seismograms observed at surface. At first, the observed receiver functions in the radial component were extracted from these seismograms using the source-equalization procedure (Langston, 1977; Owens *et al.*, 1984). Then these observed receiver functions are expected to be exactly the same as the synthetic responses on surface because the input function at the bottom of the models is equivalent to a spike function.

The result of the source-equalization procedure shows that the Langston-Owens technique can mostly recover observed receiver functions in the radial component, except at some multiples or converted P-SV phases (Figure 3). In the case of the nearly vertical incidence, ($I=1^\circ$) for example, main differences are found at the the arrival of the first multiple in which a reverse amplitude between the observed receiver functions and synthetic seismogramshas been seen at 5 sec in Figure 3a. Besides, several slight differences are seen at the later arrivals of the multiple or converted P-SV phase. These differences become even more visible when the model is more complicated. For instance, there are some reverse amplitude at the about 7 sec in Figure 3b. Such results suggest that the observed receiver functions have been contaminated by the energy of multiples that have not been properly considered in the source-equalization process.

The inverted models are also compared with the true models that are used to create the synthetic seismograms (Figure 4). The results of the inverted models depend on the complexity of the models and the incident angle of the seismic waves. For the simple model (M1), consisting of three layers over a half space, the inverted models are almost the same as the true models, and the receiver functions match well (Figures 4a and 5a). When the models are more complicated, however, the receiver functions do not match very well (Figure 5b). Furthermore, the differences between the inverted and true models (Figure 4b) become remarkable even though the general trend can be recovered. At the upper part of the models, the low velocity layer at depths from 6 to 10 km is hardly resolved. At the lower part of the models, the true models are not even limited within the error bars of the inverted models. It is believed that these mis-fits appear to have been primarily caused by the contamination of multiple energy in the observed receiver functions.

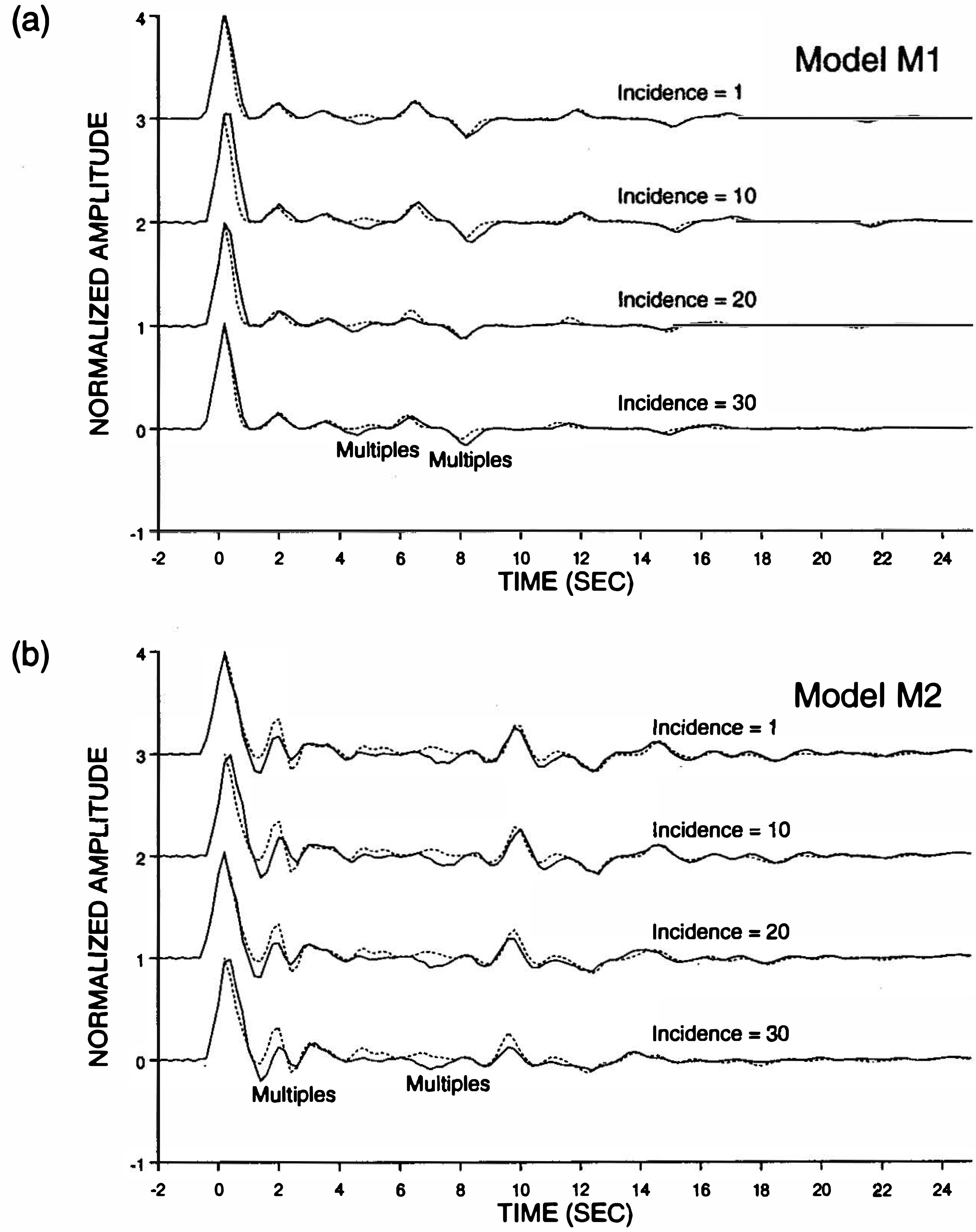


Fig. 3. Synthetic seismograms in the radial component (solid lines) generated from (a) the M1 model, and (b) the M2 model. Dotted lines show the observed receiver functions in the radial component obtained from the source-equalization procedure.

The inversion results vary significantly depending on the incident angle (Figure 4). In general, the fit of the inversion models is improved with the increase in the incident angles. One possible explanation for this feature is that when the incidence ray is close to the vertical direction, the energy of the converted P-SV phases is small, and then it is easily contaminated by the neglected multiple energy in the observed receiver functions.

3.2 Tests of the Vinnik-Kosarev Technique

The synthetic seismograms (Figure 2), used for testing the Langston-Owens technique above, are also employed for testing the Vinnik-Kosarev technique in this section. In the same way, it is expected that the receiver functions in the H-component are equivalent to

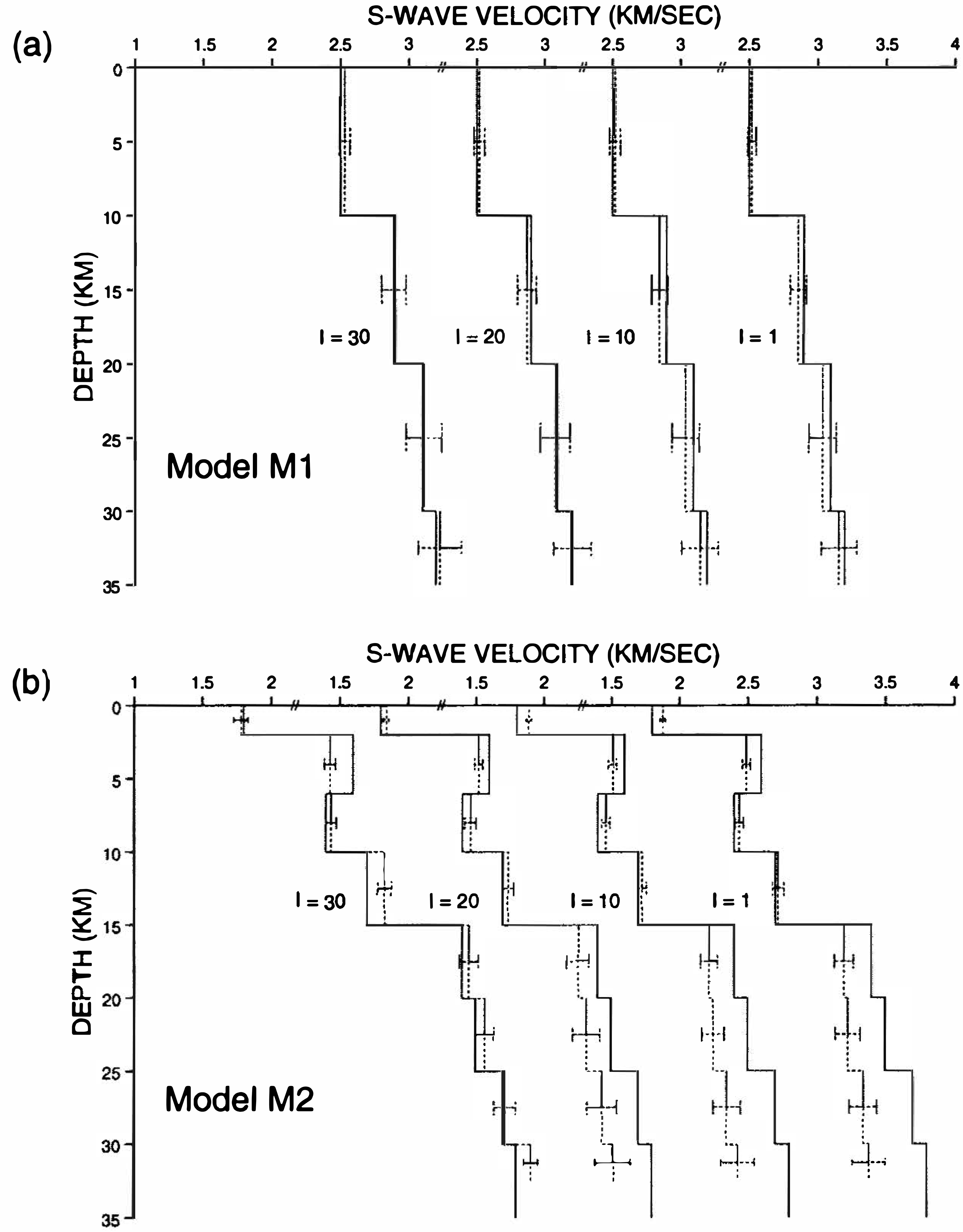


Fig. 4. True models (solid lines) and inverted models (dotted lines) got from the Langston-Owens technique: (a) the M1 model, and (b) the M2 model.

the synthetic responses on surface because the input at the bottom of the model is a spike function.

The deconvolution results in the H-component of the Vinnik-Kosarev technique can mostly recover most of the receiver functions (Figure 6). Although there are also some visible differences between the synthetic seismograms and observed receiver functions, one can not find at least any of reverse amplitudes between both seismograms. Therefore, it is believed that these results are better than those computed by the source-equalization procedure in the Langston-Owens technique.

The inversion results show that the models are generally resolved quite well even though the fit is slightly affected by the complexity of the model. For the simple model (M1), the inverted models are similar to the true ones (Figure 7a). For the complicated model (M2), the inverted results are only slightly different from those of the true models (Figure 7b).

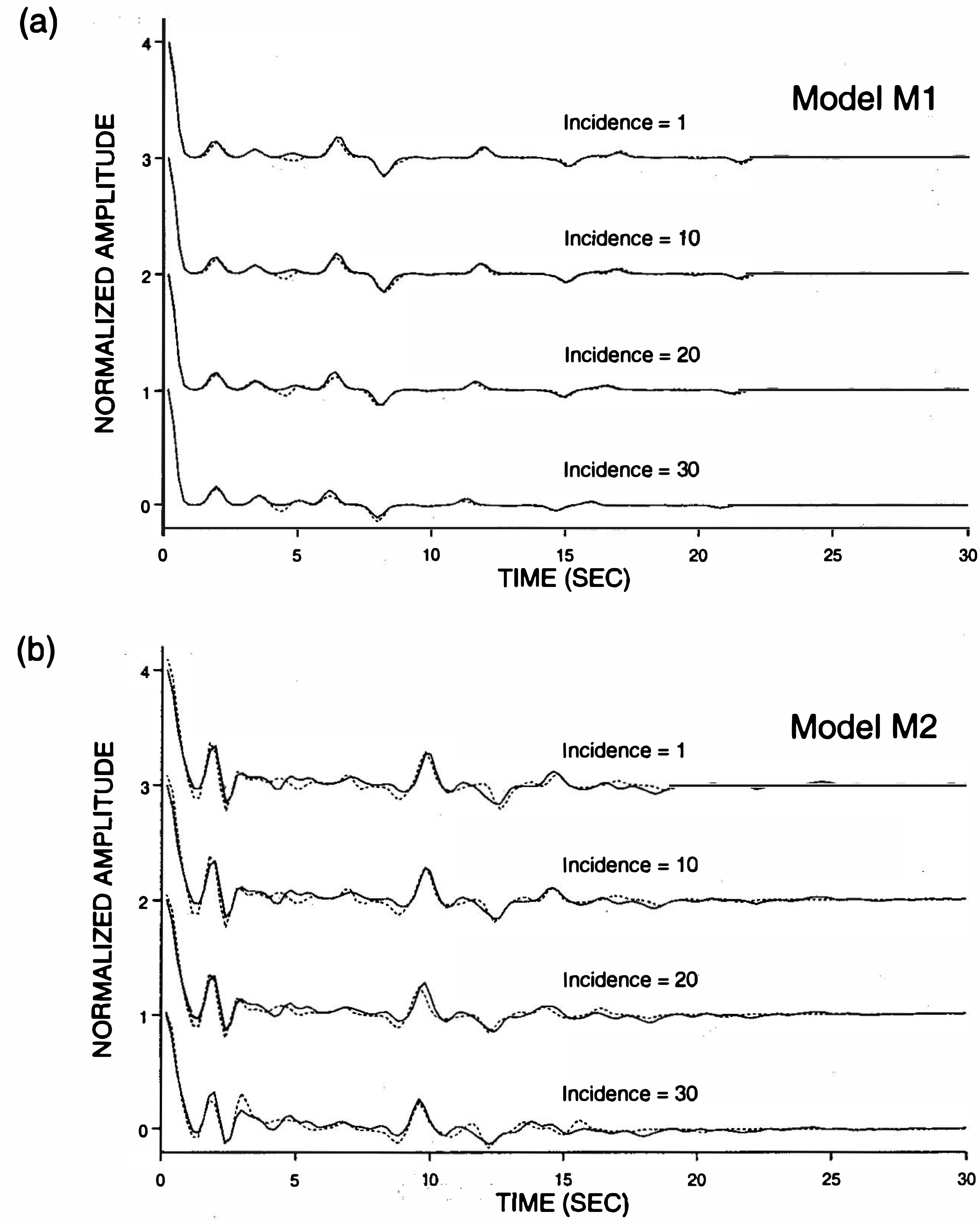


Fig. 5. Observed receiver functions in the radial component (solid lines) and calculated receiver functions (dotted lines) obtained from the source-equalization process of the Langston-Owens technique: (a) the M1 model, and (b) the M2 model.

In general, the models can be recovered not only at the low velocity layer but also at the lower part of the models. This has not been successfully resolved by the Langston-Owens technique. Therefore, it is concluded that the results computed from the Vinnik-Kosarev technique are clearly better than those from the Langston-Owens technique.

The calculated seismogram in the L-component is almost equivalent to that in the deconvolved one which is taken as the waveform of the incoming P-wave (Figure 8). In other words, the structure response in the L-component can be considered a delta function or $E_L(t)=\delta(t)$. Besides, the receiver functions in the H-component match very well (Figure 9). These results imply most energy of multiples, which has significantly contaminated the receiver functions in the Langston-Owens technique, may have already been removed or suppressed by the deconvolution processing in the Vinnik-Kosarev technique.

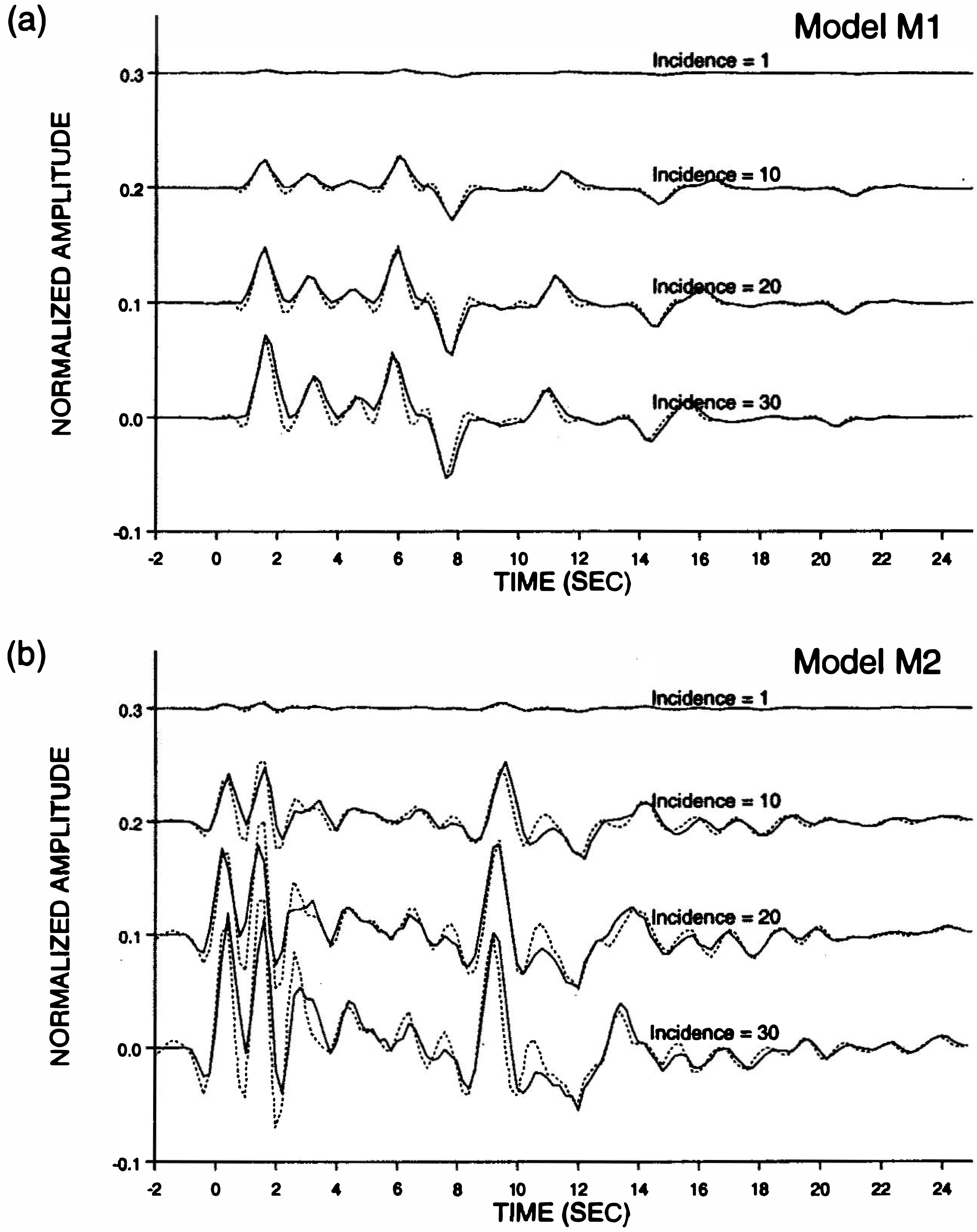


Fig. 6. Synthetic seismograms in the H-component (solid lines) generated from (a) the M1 model, and (b) the M2 model. Dotted lines show the observed receiver functions in the H-component from the deconvolution procedure of Vinnik-Kosarev technique.

A brief summary that can be drawn from the tests is that the Vinnik-Kosarev technique can mostly recover the structures from all of the test models. On the other hand, the Langston-Owens technique can successfully deal with simple structures. When the structures become complicated, care must be taken in using this technique because the calculated receiver functions for inverting the structures may be contaminated by multiples.

4. DISCUSSION

Although the results of the synthetic tests show that both the Langston-Owens and Vinnik-Kosarev techniques can mostly recover most of the true models, some visible differences have been found in their receiver functions and inverted models. In order to understand

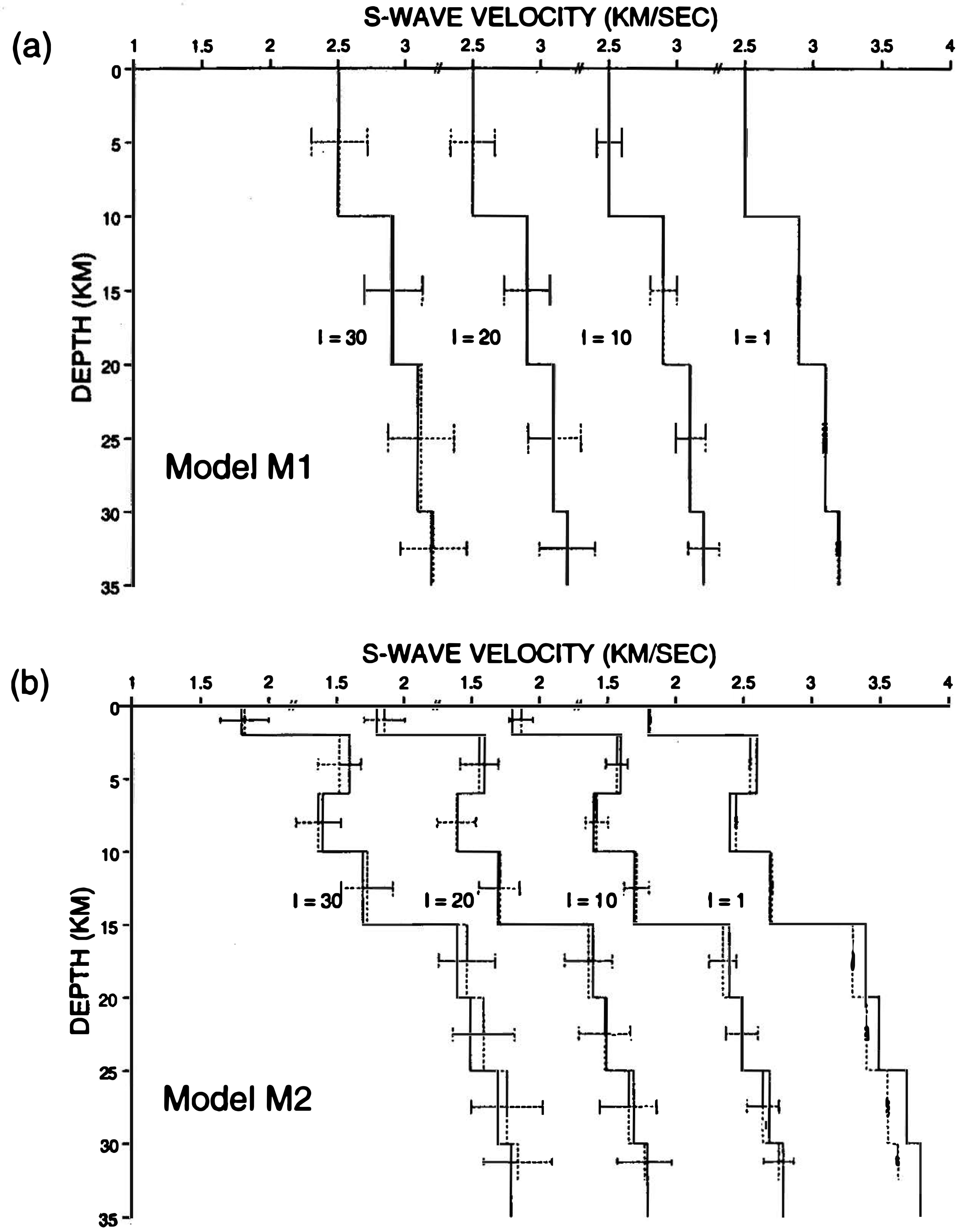
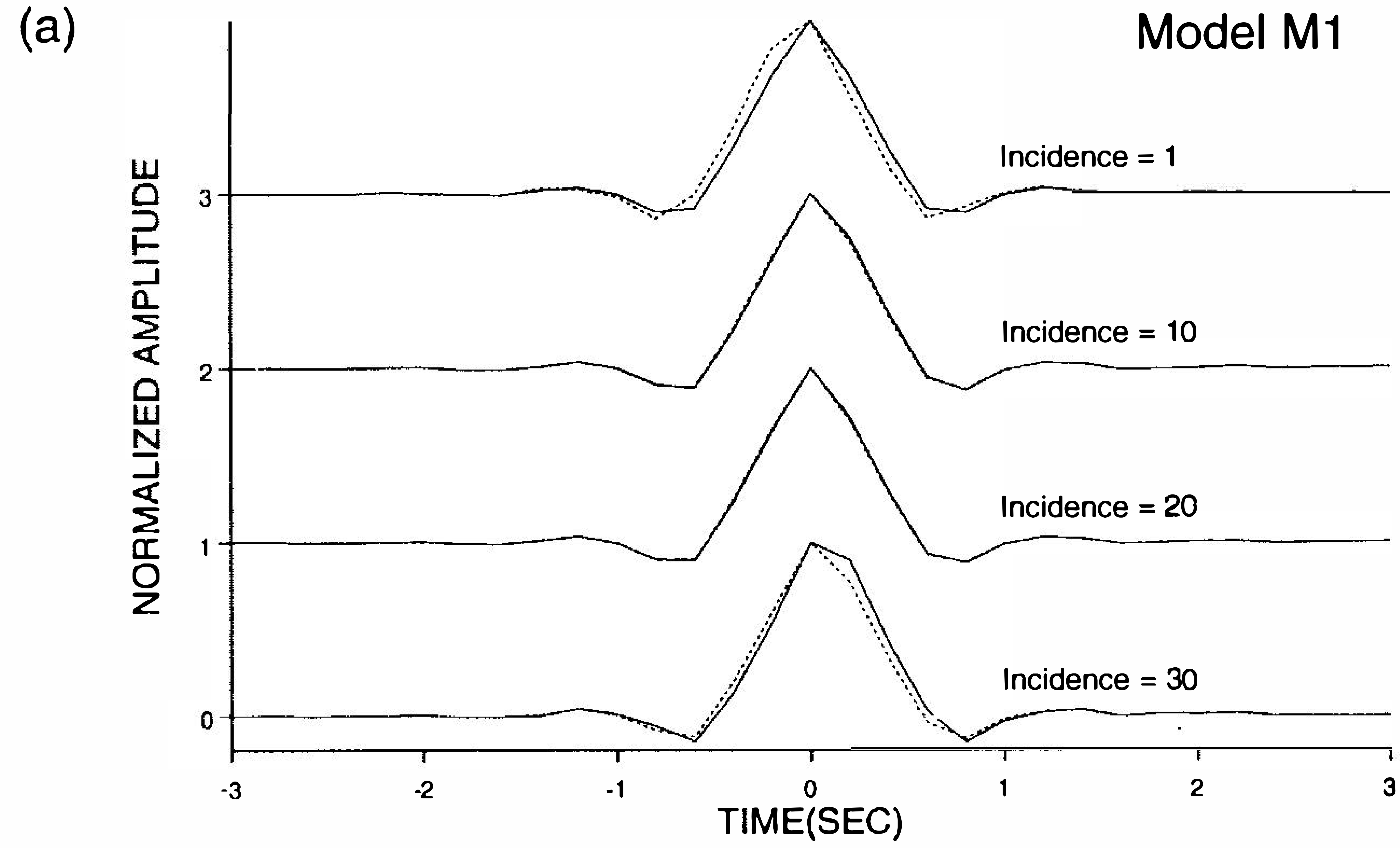


Fig. 7. True models (solid lines) and inverted models (dotted lines) got from the Vinnik-Kosarev technique: (a) the M1 model, and (b) the M2 model.

how the differences are exactly produced, some primary comparisons between two techniques are presented below. They include (1) the selection of coordinate systems, (2) the processing domain, and (3) the forward calculation.

(1) The selection of coordinate systems: Langston-Owens chose a traditional coordinate system (R and Z) to easily deal with most vertical angles of incidence. In fact, the incident angle of the teleseismic P-wave may not, however, be vertical. On the other hand, the coordinate system (L and H) chosen by Vinnik-Kosarev is more optimal in distinguishing the particle motions between the P- and SV-waves regardless of the incidence angle of the teleseismic P-waves. However, it is felt that the noticeable difference in the inversion results of the synthetic tests could hardly be caused by the selection of different coordinate systems.

(2) The processing domain: The source-equalization of the Langston-Owens technique is performed by a process of spectra division in the frequency domain. It is well known



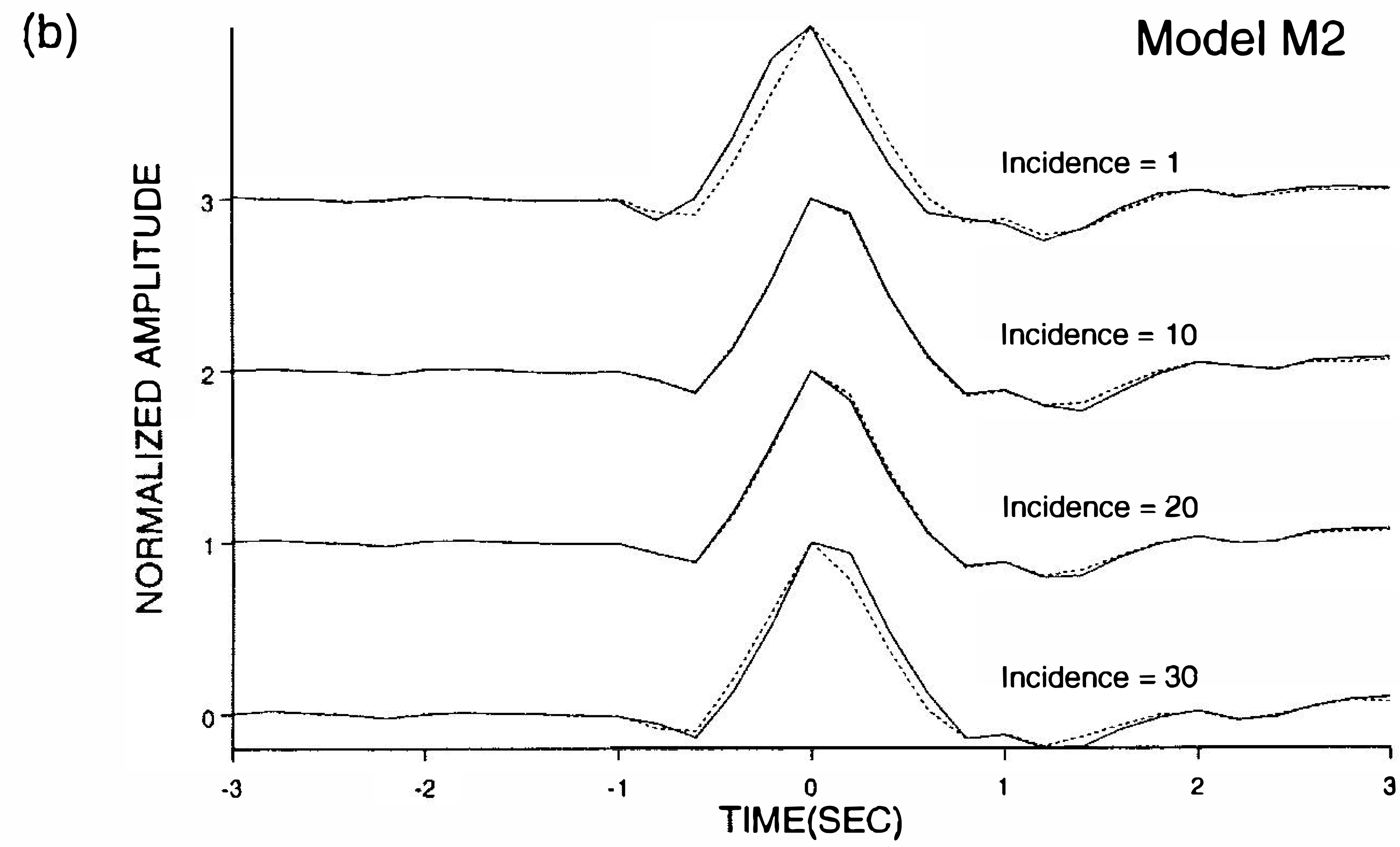


Fig. 8. Deconvolved seismograms on the L-component (solid lines) and calculated responses (dotted lines) by using the Thomson-Haskell technique: (a) the M1 model, and (b) the M2 model.

that this process is generally quite sensitive to noise in the input data. In the analysis of receiver functions, unfortunately, one of the most remarkable noises is due to multiples which are always neglected because their energy on the vertical component of steep incident P-waves is minor. In other words, it may be expected that the structural response in the vertical direction is a spike-like function or that $E_V(t)=\delta(t)$. From the synthetic seismograms, however, it has been found that the energy of crustal multiples and converted phases strongly depends on the incidence angles and the complexity of structures (Figure 2). The energy of maximum multiples can reach up to about 10-15% of the energy of the first P-wave arrival. Thus the assumption that $E_V(t)=\delta(t)$ is too simple to fit the particular cases where multiple energy is not considered as minor.

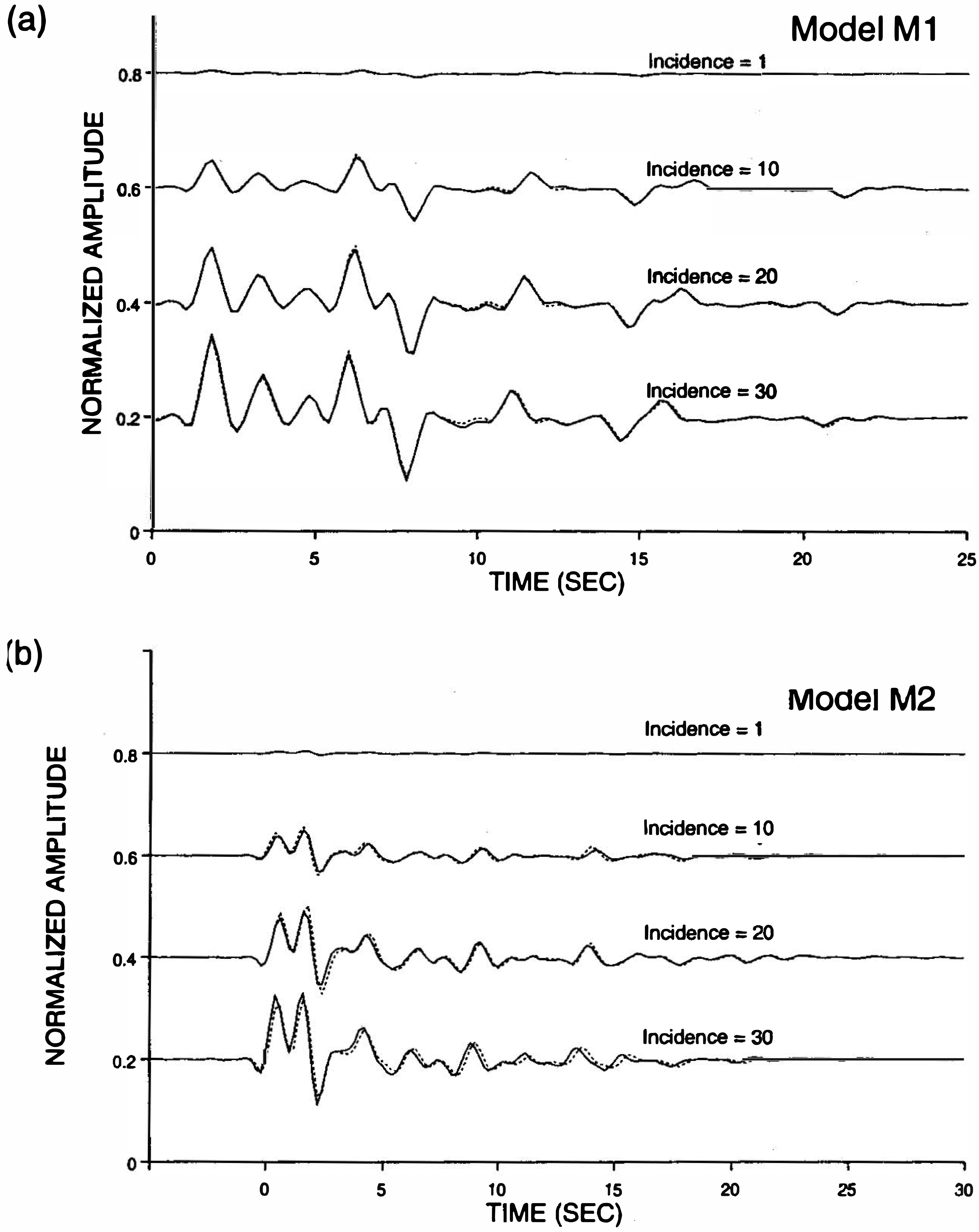


Fig. 9. Observed receiver functions on the H-component (solid lines) and calculated receiver functions (dotted lines) obtained from the Vinnik-Kosarev technique: (a) the M1 model, and (b) the M2 model.

Therefore, this simple assumption of minor multiples probably causes some problems in the receiver function analysis under the Langston-Owens technique (Figure 3).

In addition to the multiples ignored in the receiver function analyses, some high frequency noise may be introduced due to the process of discrete Fourier Transform (Brigham, 1974). For example, the sampling rate produces the aliasing problem, and the truncated length in the time domain causes rippling in the frequency domain. In the Langston-Owens technique, these noises may be filtered out by using the adjustable water-level process (or the Gaussian smoother). In the meantime, however, some original convolution signals may have truely lost all information at these frequencies where the spectra are below the water-level value.

Just like the Langston-Owens technique, the Vinnik-Kosarev technique assumes that the energy of crustal multiples is minor (Kosarev *et al.*, 1993). In the synthetic tests above (Figure 6), however, the inverted structures seemingly have not been contaminated by multiple energy because the energy of multiples has been removed or suppressed in the calculation of the deconvolution filter (a Wiener filtering) in the time domain. As a result, the inverted structures are usually better than those obtained by using the source-equalization process through the frequency domain.

(3) The forward calculation: the Vinnik-Kosarev technique uses the Thomson- Haskell method to calculate the exact receiver functions to invert the structures. To save computer time, on the other hand, the Langston-Owens technique employs a substitute method, a Gaussian smoother, to find the receiver functions. Although this substitute method does not generate the complete response of the structure, the appropriateness of this choice has been verified by comparing it with the alternative in the Thomson-Haskell method (Owens *et al.*, 1984).

5. CONCLUSIONS

The results of synthetic tests and theoretical comparisons reveal that the inversion of teleseismic waveforms to estimate the one-dimensional shear wave velocity structures beneath a seismic station can be performed equally well by both the Langston-Owens and Vinnik-Kosarev techniques if the energy of multiples on the direction of P-wave propagation is minor. However, when the structures are more complicated or multiple energy becomes stronger, the Vinnik-Kosarev technique appears to be better than the other. One of the reasons for this is the process in the different domains. The process of a source-equalization, which is performed by spectra division in the frequency domain, is very sensitive to the multiples that have been ignored in the analysis of the receiver functions. On the other hand, the process of a deconvolution filtering in the time domain can successfully handle multiples. In any case, both techniques consistently indicate that the inversion results are strongly dependent upon the incident angle of a seismic wave. In fact, for the nearly vertical incidence, neither technique can resolve the models very well because the energy of the converted P-SV phases is very small.

Acknowledgements The author would like to thank Prof. Steve W. Roecker who has had a significant influence on this work. Special acknowledgement is extended to Grisha Kosarev and Tom Owens who allowed the author to use parts of their computer programs.

REFERENCES

Berkhout, A. J., 1977: Least-squares inverse filtering and wavelet deconvolution. *Geophysics*, **42**, 1369-1383.

Brigham, E. O., 1974: The Fast Fourier Transform. Englewood Cliffs. New Jersey. 252pp.

Dey-Sarker, S. K., and R. A. Wiggins, 1976: Upper mantle structure in western Canada. *J. Geophys. Res.*, **81**, 3619-3632.

Haskell N. A., 1962: Crustal reflection of plane P and SV waves. *J. Geophys. Res.*, **67**, 4751-4767.

Kosarev G. L., L. I. Makeyeva, L. P. Vinnik, 1987: Inversion of teleseismic P-wave particle motions for crustal structure in Fennoscandia. *Phys. Earth Planet. Inter.*, **47**, 11-24.

Kosarev G. L., N. V. Peterson, L. P. Vinnik, and S. W. Roecker, 1993: Receiver functions for the Tien Shan analog broadband network: Contrasts in the evolution of structures across the Talasso-Fergana fault. *J. Geophys. Res.*, **98**, 4437-4448.

Langston, C. A., 1977: The effect of planar dipping structure on source and receiver responses for constant ray parameter. *Bull. Seis. Soc. Am.*, **67**, 1029-1050.

Langston, C. A., 1979: Structure under Mount Rainier, Washington, inferred from teleseismic body waves. *J. Geophys. Res.*, **84**, 4749-4762.

Owens, T., G. Zandt, and S. R. Taylor, 1984: Seismic evidence for an ancient rift beneath the Cumberland Plateau, Tennessee: A detailed analysis of broadband teleseismic P waveforms. *J. Geophys. Res.*, **89**, 7783-7795.

Owens, T., 1987: Crustal structure of the Adirondacks determined from broadband teleseismic waveforms. *J. Geophys. Res.*, **92**, 6391-6402.

Vinnik, L. P., 1977: Detection of waves converted from P to SV in the mantle. *Phys. Earth Planet. Inter.*, **15**, 39-45.