

Characteristics of Reflectivity Strength on a Thin Bed

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ABSTRACT

From the point of view of resolution, an existing thin layer may not be detected by seismic wavelets. By numerical experiment and using the reflectivity strength, it is illustrated that the existence of a thin layer with a thickness of less than one-eighth of the dominant wavelength of the propagating seismic wavelet can be detected.

An observed seismic wavelet consists of subsurface reflectivity, i.e. the composite wavelets are a function of the separations of individual reflectivity alone. For a thin layer, the shape of a composite seismic wavelet is a function of layer thickness. Using plane wave theory and assuming no energy is dispersed, the authors calculate a synthetic seismogram for a geologically pinchout model based on an input Ricker wavelet. The calculated composite wavelets are then cross-correlated with the derivative of the input wavelet. From Widess's (1973) studies, the resolvable ability of a seismic wavelet is clearly defined and understood by the correlation. To examine the effects of the thickness of a thin layer on reflectivity strength, the Hilbert transform is then used to transfer synthetic wavelets. By destructive interference, reflectivity strength shows a minimum when the layer thickness is less than one-eighth of the dominant wavelength of the wavelet. The minimum no longer occurs as the layer thickness exceeds the above criterion. This phenomenon of reflectivity strength on the layer thickness of a "real" thin layer can be considered as an indication of its existence.

(Key words: Cross-correlation, Resolution, Thin layer, Reflectivity strength)

1. INTRODUCTION

Seismic signals recorded at the surface carry subsurface geological information as the wave propagates. With the continuity/discontinuity of the observed signals in a reflection seismogram, the features of this subsurface structure are reconstructed and interpreted. Consequently, on the base of the attributes of the recorded wavelet, the potential of hydrocarbon resources can be evaluated.

From the perspective of sedimentary and tectonic processes, most geological structures

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have a smaller vertical dimension than a horizontal one. Seismically, for a geological reservoir the ratio of the vertical dimension to the horizontal plays a very important role in reflection exploration. As this dimensional ratio of a geological event decreases, the difficulty of identifying the event increases.

The structure of an "unobvious" dimensional ratio is treated as a thin layer in reflection seismology. The existence of thin layers themselves exhibits a special meaning to an exploration geophysicist. In the history of petroleum exploration, structures of an "obvious" dimensional ratio (e.g. anticlines, faults, or domes etc.) have been widely explored and become depleted and exhausted. Therefore, locating and detecting the existence of hydrocarbon structures of an "unobvious" dimensional ratio (e.g. pinchout, on-lap, off-lap, lenticular sand etc.) are gaining more and more importance. However, to resolve and detect a thin layer are not only difficult but also challenging.

To date, most of the research done on a thin layer concentrated on analyzing how an observed wavelet is distorted by the effects of the boundaries of a thin layer. This has been studied in detail with the conclusion that the limitation of the vertically separated reflectivity to be resolved is dominated by the wavelength of the propagating wavelet by which the composite wavelet is observed. Widess (1973) pointed out that the power of resolvability of a thin layer is frequency dependent, i.e. wavelength dependent. By convoluting a zero-phase wavelet with two spikes of equal amplitude but opposite polarity, Widess observed that as the separations between spikes decrease to one-eighth of the dominant wavelength of the propagating wavelet, a "stable" composite wavelet occurs. This stable composite wavelet has its semblance similar to the derivative of the wavelet convoluted. The separation of one-eighth of a wavelength is thus defined by Widess as a criterion by which the adjacent interfaces, top and bottom boundaries, of a thin layer could be resolved. He also concluded the magnitude of a composite wavelet is approximately proportional to the thickness of a thin layer. Meanwhile, constructing different combinations of top and bottom reflectivity of a thin layer, Meissner and Meixner (1973) investigated the shape of the wavelets which have been distorted and derived the similar results.

Thus, the shape of a composite wavelet has been thoroughly analyzed and has provided information for identifying the reflections of the upper and the lower boundaries of thin layers. Studying the shape of a composite wavelet does address lots of remarkable contributions in thin layer resolution. Instead of dealing with resolving reflections from thin layers, the present authors concentrate on detecting the existence of the layers. The response of reflectivity strength and instantaneous amplitude obtained by transforming a seismic trace using Hilberts technique on a composite wavelet are studied.

2. NUMERICAL MODEL

To generate a synthetic seismogram of a geological pinchout, a zero-phase Ricker (1940) wavelet is calculated to convoluted with spikes of equal amplitude and opposite polarity. The mathematical expression of the wavelet is

$$f_R(t) = A_j (1 - 2\pi^2 v_M^2 t^2) \exp[-\pi^2 v_M^2 t^2] \quad (1)$$

where A_j is the peak amplitude, and v_M is the peak frequency of the amplitude spectrum of

the wavelet. The calculated zero-phase Ricker wavelet and its derivative are shown in Figure 1. The ν_M of the Ricker wavelet is 50 Hz. The peak amplitude, A_i , is arbitrarily set at 100.

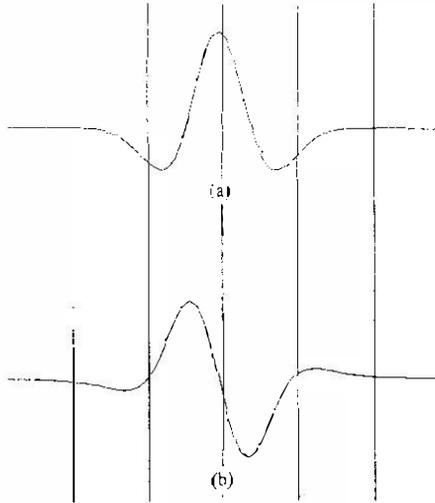


Fig. 1. (a). Ricker wavelet computed by Eq. (1). The peak frequency of the wavelet is 50 Hz. (b) The derivative of (a).

Assuming no absorption and no transmission loss, composite wavelets, with a zero separation between spikes to one wavelength (period), are calculated. The mathematical form of the composite wavelet, with amplitude A and a separation increment of $\lambda/16$ (or $T/16$), can be written as:

$$f_c(t) = A(1 - 2\pi^2\nu_M^2 t^2) \exp[-\pi^2\nu_M^2 t^2] + A(1 - 2\pi^2\nu_M^2 (t + n\Delta t)^2) \exp[-\pi^2\nu_M^2 (t + n\Delta t)^2], \quad (2)$$

$$\text{where } \Delta t = \frac{T}{16}, \quad T: \text{period} \quad n = 1, 2, \dots, 16.$$

A synthetic seismogram is calculated at a 1-ms sampling interval and is displayed in Figure 2. There are eighteen synthetic traces in Figure 2. Trace 1 is the composite wavelet of zero separation (i.e. diffraction). The trace on the left hand side of Trace 1 is its derivative form (Figure 1 for comparison). The separation between spikes in Trace 3 is $2\lambda/16$; $3\lambda/16$; for Trace 4; and $4\lambda/16$ for Trace 5. The coherence (similarity) of the derivative trace and Trace 3, 4, or 5 are visible. The symbol, λ , stands for the dominant wavelength of a propagating seismic wave of the analyzed wavelet.

Figure 3 shows the cross-correlation of the derivative wavelet and the other composite wavelets (Traces 1~17) in the synthetic seismogram (Figure 2). According to constructive interference, it is expected that the maximum coefficient of the correlation should occur at the separation of $\lambda/16$ (Trace 5) between spikes; however, it seems this is not the case in the present computation. On the contrary, instead of Trace 5, a maximum appears at Trace 4. This means the separation between spikes is $3\lambda/16$. However, this may be easily explained. The expected result would occur only when two sinusoidal wavelets of equal amplitude and opposite polarity interfere, but here the source wavelet by which the seismogram is derived and processed is a *zero-phase Ricker wavelet*.

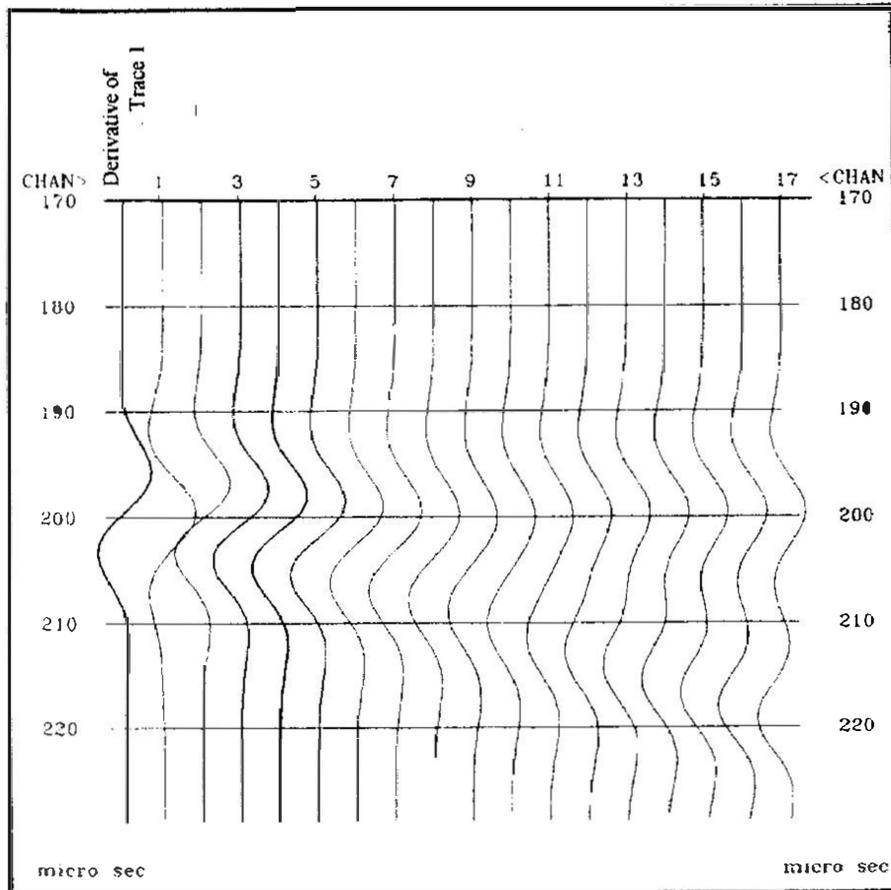


Fig. 2. Composite wavelets calculated by convoluting a zero-phase Ricker wavelet with two spikes of equal amplitude and opposite polarity with 1-ms as a sampling interval. The derivative of the source wavelet is shown to the left of Trace 1. The separation of the spikes varies with an interval of $\lambda/16$ and that of Trace 1 is zero. A comparison of the derivative wavelet should be made to Trace 3, 4 and 5 (separations are $2\lambda/16$, $3\lambda/16$ and $4\lambda/16$ respectively). A similarities can be seen.

From Figure 3, it can also be seen that the normalized magnitude of the cross-correlation is about 0.8 at a separation of $\lambda/8$ (Trace 3). When the results are compared to similar research done by Ricker (1953), Widess (1980) and Kallweit (1982), a coincidence is found. The reflections of a composite wavelet are resolvable only if their separation exceeds one-eighth of the wavelength of the analyzed wavelet. However, due to inherent complexities, if the wavelet is deformed, this criterion might change: in other words, the minimum resolvable separation might increase.

3. REFLECTIVITY STRENGTH ANALYSIS

To discuss and verify the criterion of the resolvable limitations is not the purpose of this study. The objectives here are to investigate the effects of layer thickness on reflectivity strength and to detect the existence of a thin layer. Among the attributes of seismic data that are considered for stratigraphic interpretation, amplitude is the most frequently adopted. Bright spot has been considered an indication of hydrocarbon resources; amplitude versus offset (AVO) has been widely analyzed to study the variation of subsurface lithology. To access more information from the amplitude of a seismic wavelet, a seismic trace is Hilbert transformed and the relationship between reflectivity strength and thickness of a thin layer is investigated.

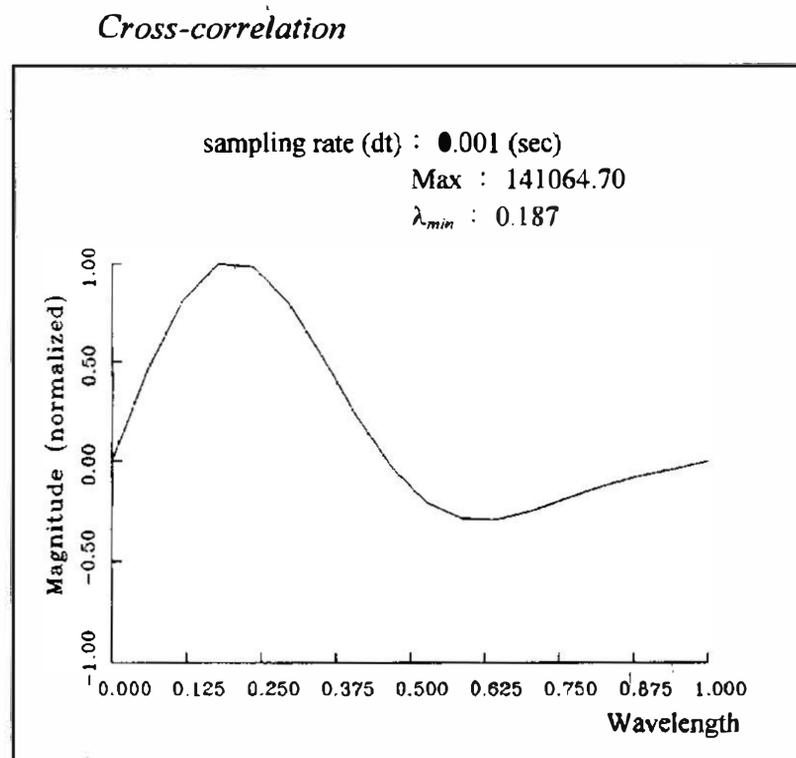


Fig. 3. Cross-correlation of the derivative wavelet (source wavelet) and composite wavelets (Traces numbered 1 to 17 in Figure (2)). At zero spike separation the correlation coefficient is zero. The maximum appears at the separation of $3\lambda/16$ (Trace 4) for an input Ricker wavelet.

3.1 Theory Review

The Hilbert transform of a function $f(t)$ is defined as:

$$H(f(t)) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(\tau)d\tau}{\tau - t} \quad (3)$$

by Bracewell (1986). Mathematically, Eq. (3) can be considered as a convolution. The equivalent expression of Eq. (3) in convoluted form is

$$H(f(t)) = \frac{-1}{\pi t} * f(t) \quad (4)$$

The application of two Hilbert transforms in succession reverses the phases of all components;

$$f(t) = -\left[\frac{-1}{\pi t}\right] * H[f(t)], \quad (5)$$

$$f(t) = -\frac{1}{\pi t} \int_{-\infty}^{\infty} \frac{H[f(\tau)]d\tau}{\tau - t}$$

If the Hilbert transform pair, $H[f(t)]$ and $f(t)$, is itself Hilbert transformed, the resulting pair becomes $-f(t)$ and $H[f(t)]$. This polarity reversal is simply a result of $\pi/2$ phase advances. Hence, a seismic trace $f(t)$ can be defined as the real part of an analytic trace $F(t)$, and the $f^*(t)$ is the quadrature series.

$$F(t) = f(t) + if^*(t) = A(t)[\cos \theta(t) + i \sin \theta(t)], \quad (6)$$

$A(t)$ is amplitude spectrum.

The scheme of the simplified discrete Hilbert analysis for computer programming is described as follows:

$$\begin{aligned} f_a &\rightarrow F_0, F_1, F_2, \dots, F_{n/2}, \dots, F_{n-1}, \rightarrow f_\alpha, \quad \text{Fourier transform} \\ \text{multiply } &1, 2, 2, \dots, 1, 0, \dots, 0 \rightarrow H_\alpha \quad \text{Hilbert transform} \end{aligned} \quad (7)$$

where

$$\begin{aligned} \text{real}(H_\alpha) &\equiv f_\alpha = f(t), \quad \text{and} \\ \text{imag}(H_\alpha) &\equiv f_\alpha^* = \text{quadrature series of } f(t). \end{aligned}$$

$F_{n/2}$ is the Nyquist frequency of $f(t)$. Once the data is transformed, some instantaneous seismic attributes can be readily obtained. The instantaneous attributes associated with the transformed wavelet include: instantaneous amplitude, instantaneous phase, and instantaneous frequency.

With $f(t)$ and $f^*(t)$ calculated, the reflectivity strength of the correspondent instantaneous amplitude, $A(t)$, and the instantaneous phase, $\theta(t)$ are directly obtained by performing the following manipulations:

$$A(t) = \sqrt{\text{real}(H_\alpha)^2 + \text{imag}(H_\alpha)^2} = \sqrt{f^2(t) + f^{*2}(t)} = |H_\alpha(t)|, \quad (8)$$

and instantaneous phase

$$\theta(t) = \tan^{-1} \left[\frac{\text{imag}(H_\alpha(t))}{\text{real}(H_\alpha(t))} \right] = \tan^{-1} \left[\frac{f^*(t)}{f(t)} \right]. \quad (9)$$

Among all of the attributes derived, instantaneous amplitude measures the reflectivity strength, which is proportional to the square root of the total energy of the wavelet at an instant time. Instantaneous phase measures the continuity of the events on a seismic section. Instantaneous frequency is computed from the temporal rate of change of instantaneous phase (Taner, *et al.*, 1979).

Studying the instantaneous attributes obtained by Hilbert transformation helps in interpreting seismic data from different points of view perspective. To obtain more detailed information about how the reflectivity strength of composite wavelets vary with layer thickness, the increment of layer thickness for the successive composite is found and adjusted to $\lambda/32$. The curve of the variation of reflectivity strength with layer thickness is shown in Figure 4.

In Figure 4, it should be noted that the magnitude of reflectivity strength decreases right after zero-separation and jumps to a minimum. The minimum shows up at the layer thickness of 0.0313λ ($\lambda/32$), where the maximum destructive interference occurs. It can also be seen that as the separation of the layer boundaries increases and exceeds 0.125λ ($\lambda/8$), the effects of destructive interference may no longer be that obvious. The response of reflectivity strength of composite seismic wavelet which is exhibited on the thickness of a thin layer is now clearly demonstrated and understood. The occurrence of the minimum in reflectivity strength for a thin layer thickness less than its resolvable thickness (criterion) can be considered an indication for its existence.

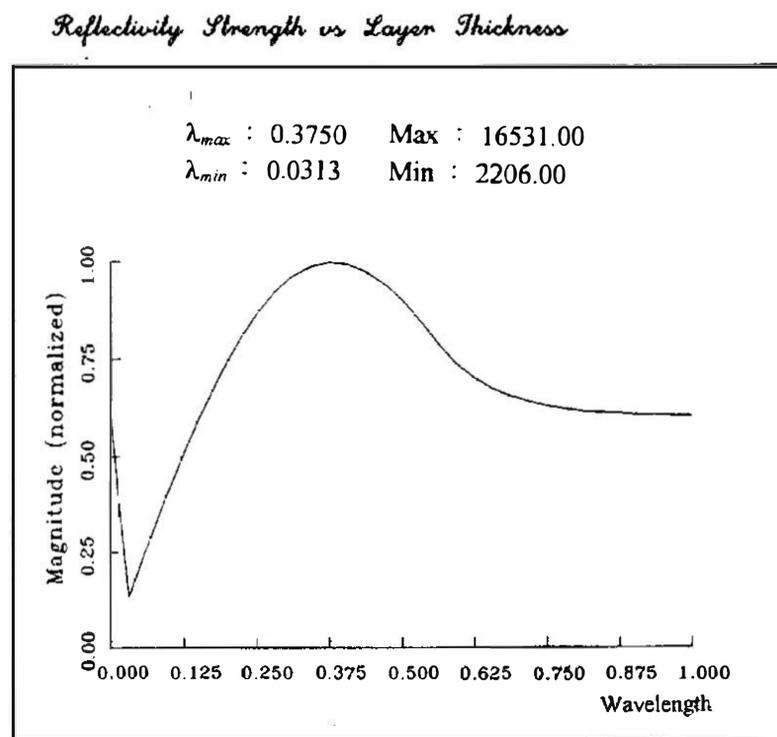


Fig. 4. Graph of a Hilbert envelope derived from numerical synthetic data. Note, the separation in successive trace computations has been adjusted to $\lambda/32$. A minimum in the reflectivity strength shows up at $\lambda/32$ in spikes separation. The minimum indicates the existence of a thin layer. For a coarser space sampling interval exceeding $\lambda/8$, the minimum disappears, and successfully adopting this criterion for analysis may no longer apply.

4. DISCUSSION AND CONCLUSIONS

A thin layer, formed by the intrusion or sedimentary process, commonly exhibits opposite reflectivity. If the separation of the layer boundaries is large enough, say exceeding one-eighth of the dominant wavelength of the propagating wavelet, and if it can be resolvable, the volume of the layer can be estimated by analyzing the configuration of composite wavelets. Nevertheless, a thin layer of unresolvable thickness will very possibly be ignored and become invisible on the seismic section.

In this research, a synthetic seismogram is calculated using a Ricker wavelet as a source wavelet to convolute with a geological pinchout model of equal reflectivity but opposite polarities. Based on the physical properties of wavelet interference, a composite wavelet is studied both to understand the resolvability of the wavelet for an existing thin layer and to investigate the reliability of the reflectivity strength of the wavelet in detecting the existence of a layer.

To see the resolution of a composite wavelet, which is formed by the interference from reflections from the top and bottom of a thin layer, a Ricker wavelet is generated and differentiated. For interfered reflections of equal amplitude and opposite polarity, the shape of the composite wavelet converges into the derivative shape of an input wavelet at the layer thickness of one-eighth of the dominant wavelength of the propagating wavelets (Widess, 1973). The similarity between the derivative of input (source) wavelet and the composite wavelet provides a criterion for the resolvability of the reflections. Thus, to resolve a composite wavelet in accordance with the notion of Widess idea, the correlation technique works rather successfully.

However, to detect the existence of a "real" thin layer with a layer thickness of less than $\lambda/8$, the correlation technique can be no longer be applied. On the other hand, reflectivity strength analysis works more efficiently. For a "real" thin layer, reflections of opposite polarization occur and mingle at its boundaries. These results show the existence of the "real" thin layer can be revealed by reflectivity strength analysis. In short, the thinner the layer is, the more exaggerated is the sensitivity. Hence, the sensitivity of reflectivity strength responds to layer thickness of the layer can be adopted with confidence to locate a "real" thin layer.

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