Synthetic Seismogram for the Velocity and Attenuation Structure Near the Inner-outer Core Boundary Using the Generalized Ray Method

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ABSTRACT

Synthetic seismograms for seismic waves penetrating near the innerouter core boundary are implemented using the generalized ray method. The model takes into account the detailed velocity jump near the innerouter core boundary and the possible depth dependent attenuation of the inner core. The source-side surface reflections, considered as the later phases of the observed seismograms, are included in the model. The program developed in this study is suitable for the modeling of shallow and intermediate deep events along the major seismic zones and the mid-ocean ridges. These areas provide good global coverage of ray paths for studying the structure of the earth's core in detail. Numerical modeling in this study shows that the source-side surface reflections of the core phases make a significant contribution to determining the core velocity structure for the observations of the spatial-dense array with small aperture. Additionally, the possible depth dependent inner-core attenuation can be resolved from regional array seismograms. The results of this study show that the newly developed generalized ray code displays a high potential for further elucidating the earth's core structure using the available data.

(Key words: Generalized ray method, Synthetic seismogram, Inner-core boundary, Attenuation)

1. INTRODUCTION

One of the major objectives in examining the velocity structure of the inner core is to understand the evolution and dynamics of the earth's core. The outer core is composed of an iron alloy with nickel and light elements such as sulfur and silicon. At the inner-outer core boundary (ICB) iron precipitates upon the inner core, thus releasing the light elements to the outer core. It is widely accepted that the chemical differentiation provides a large part of the energy driving the earth's dynamo (Loper and Roberts, 1981). In addition to the velocity structure, the attenuation of seismic waves in the inner core often distinguishes important con-

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straints on core dynamics and on the inner core freezing processes. It has been proposed (Loper and Roberts, 1981) that the continuous solidification of iron from a liquid-core alloy may result in a high attenuation mushy zone at the ICB. This mushy zone could be very thick, even possibly extending into the center of the earth. To investigate the inner core, seismic observations can provide direct evidence of its properties. This is because both the velocity and attenuation structure of the inner core could be responsible for the propagation of seismic waves which penetrate into the inner core. In the past, the first arrival times and amplitudes of the core phases were frequently used to estimate the earth's core structure (Hage, 1983; Johnson and Lee, 1985) based on geometrical ray methods (Julian and Anderson, 1968; Buland and Chapman, 1983; Huang 1996a). However, due to the limitations of global seismicity and of the station distribution of the World-Wide Seismic System Network (WWSSN), the ray coverage through the inner core was ill-constrained to invert them. Presently, the global digital seismic network (GDSN) is installed with the broadband instruments to monitor global seismicity and to provide high quality seismic waveform data for the study of the deep interior of the earth. The inner core structure is probably improved by studying multiple core phases, which penetrate different parts of the core, through waveform modeling in one station (Figure 1). Furthermore, some regional seismic arrays, such as the German Regional Seismic Network (GRSN), the Canadian National Seismograph Network (CNSN) and the TERRAscope array in southern California, also include high quality digital array waveforms for the earth core study. These data are easily retrieved via the Internet.

Several methods have previously been proposed to synthesize seismic waveforms for core structures. These include the reflectivity method (Muller, 1985), the WKBJ method (Chapman, 1978), the full-wave method (Cormier and Richards, 1977) and the generalized ray



Fig. 1. Ray paths of seismic waves near the ICB. The path AB goes through the top of the outer core, BC goes through the bottom of the outer core (OC) and DF refracts from the inner core (IC).

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method (GRT) (Helmberger, 1983). The synthetic seismograms from these different methods have been confirmed as being quite similar (Burdick and Orcuut, 1979). Even so, one particular advantage of the GRT is its ability to isolate contributions associated with specific paths which are important to core structures (Song and Helmberger, 1992). However, the GRT used by Song and Helmberger (1992) was simplified for explosions or deep events only. With their approach, the source-side reflections were excluded from the model. Using deep earthquake data alone to study the earth's core, the global coverage was ill-constrained because of the exclusion of shallow events from mid-ocean ridges and major seismic zones. Furthermore, Song (1994) took the inner-core attenuation as a constant quality factor (Q-value) or constant t* (travel time/ average Q). However, recent studies for the inner-core attenuation (Niazi and Johnson, 1992; Bhattacharyya *et al.*, 1993; Souriau and Roudil, 1995) have shown the complex Q structure near the ICB. Clearly, it is unlikely that a uniform Q model can reliably explain observations of present inner-core Q studies. Thus, it is necessary to explain inner-core attenuation by its lateral variations.

In this study, the above limitations of Song (1994) were overcome. The new extension included both the extra rays from the source side boundary reflections to model the core phases, and the depth dependent Q structures. The modeling results of this study showed that the pPKP and sPKP (surface reflected PKP waves by the P- and S-waves from source, respectively) made a significant contribution in determining the velocity gradient near the ICB. The newly developed GRT program offered greater flexibility in selecting data, especially those from a spatial-dense array with small aperture with which to study the core structure and its anelastic properties. The waveform model also verified the suitable resolution from which the depth dependent inner-core Q structure could be determined.

2. THEORY

Synthetic seismograms based on the generalized ray and the Cagniard-de Hoop method has been used in previous modeling of effects at the core-mantle boundary (CMB) and the ICB and has been found to be quite useful (Lay and Helmberger, 1983; Song and Helmberger, 1992). The response of the earth models at ranges beyond the shadow zone can be obtained by simply summing primary rays. That is:

$$G(t) = \sqrt{\frac{2}{r}} \frac{1}{\pi} \sqrt{\left(\frac{\Delta}{\sin\Delta}\right)} \left[\frac{1}{\sqrt{t}} \cdot J(t)\right],\tag{1}$$

where

$$J(t) = \sum_{k=2}^{n} Im \left(\frac{\sqrt{p}}{\eta_1} \left(\frac{dp}{dt} \right)_k \prod_k (R, T) A_k(t) \right), \tag{2}$$

and p is the ray parameter; r is the radius of the earth; η is (1/v² - p2); v is the velocity of layer

medium; Δ is the epicentral distance in degrees; Σ is the summation of contributing rays; A is the attenuation operator (Ruff and Helmberger, 1982); and Π is the product of the transmission and reflection coefficients. The primary rays refer to those that undergo one reflection only. The $\Delta/\sin \Delta$ factor is a correction for spherical spreading (Gilbert and Helmberger, 1972). Since the ray summation omitted multiples, it appears necessary to document these responses by comparing them with those of other methods. This was done by Song and Helmberger (1992) who compared the GRT and the full-wave (Cormier and Richards, 1977) methods with similar waveforms. In the work by Song (1994) it was assumed that the attenuation factor was nearly the same as for the reflection depth range of k=2 to n of Equation (2). There, A(t) was taken outside the sum, while the attenuation operator with a single value for t^* was convolved with the total elastic response (J(t)) of Equation (2) and excluded A(t). However, if the attenuation varies significantly over the depth range of interest, it is necessary to convolve a different t^* operator with each individual generalized ray. This is the same as in Equation (2) above. The theory and implementation for the layered O modeling for studying the O structure near the core-mantle boundary region was discussed in detail by Ruff and Helmberger (1982). In this study, their approach was extended to model the seismic rays penetrating into the earth's core to study the Q structure near the ICB. Following their representation, the vertical response of the core phases under consideration for a dislocation source could be generated as:

$$W(t) = \frac{M_o}{4\pi\rho_o} \left(\frac{d}{dt} \left[\left(D(t) \right) \bullet \sum_{j=1}^3 A_j(\theta, \delta, \lambda) G_m(t) \right] \right].$$
(3)

where

• is the convolution operator; $A_j(\theta, \delta, \lambda)$ is the radiation pattern; D(t) is the source time function; and

$$G_{m}(t) = C_{j}^{D}(p_{o})R_{PZ}(p_{o})G_{PKP}(t) + C_{j}^{U}(p_{o})R_{pPZ}(p_{o})G_{pPKP}(t) + (SV_{j}^{U})(p_{o})R_{SPZ}(p_{o})G_{sPKP}(t)$$

is Green's function which includes the response of the PKP phase (G_{pKP}) and its surface P- and S-wave reflections (G_{pPKP}) and G_{sPKP} , respectively. C^U, C^D and SV^U are the vertical radiation of P, pP and sP, respectively. R_{NZ} indicates the appropriate receiver function for either PKP (N=P), pPKP (N=PP) or sPKP (N=SP) waves arriving at the station, respectively. In modeling the waveforms of an explosion or deep events, only G_{pPKP} of $G_m(t)$ were considered by Song (1994), and C^D was assigned as the unit. In this study, to accurately obtain the core phases near caustic, Green's function of each core phase in the same epicentral distance was computed individually, and the vertical radiations were estimated according to ray parameters. The synthetic seismograms of the core phases were finally obtained by the summation of the individual core phase responses.

3. MODELING RESULTS

A. Comparison with other results:

To check the accuracy of the programming, in this study, a test with other verified results was necessary. The comparison of the synthetics from this study with the computations by Song (1994) showed a good agreement. A simple comparison is given in Figure 2. Both seismograms were computed using the PREM model (Dziewonski and Anderson, 1981) with a source depth of 600 km and an epicentral distance of 159°. To smooth out high-frequency numerical noise, both seismograms have been convolved with a one second triangular source time function (Song, 1994). Both travel time and amplitude were found to consistent. The slight difference in the tail of the PKP(DF) phase may have been due to the truncation errors from Cagniard-de Hoop contour integration.



Fig. 2. Comparison of the seismic waveforms computed in this study with those by Song (1994) from individually developed generalized ray codes. Both seismograms were computed for a station on epicentral distance of 159° from the PREM model with source depth 600 km. The zero time of both traces was set as 1114 seconds after earthquake initial time.

B. Modeling waveforms for standard earth models:

The amplitudes and waveforms of core phases are sensitive to the ICB structure. In this study, the synthetic seismograms of the four often used earth models, namely, the PREM, IASP91 (Kennett and Engdahl, 1991), SP6 (Morelli and Dziewonski, 1993) and the AK135 (Kennett *et al.*, 1995), were modeled to show the sensitivity of the waveform variations. Near the ICB, the P-wave velocity structures of these models were slightly different, as shown in Figure 3. The synthetic waveforms computed by the GRT are shown in Figure 4. It can be seen that the relative amplitudes and travel times of core phases from different branches did in fact vary. The detailed analysis for these phases and comparison with the standard earth models and a newly developed earth core model, PREM2 (Song and Helmberger, 1995), will provided useful information for determining the lateral variation of the ICB structure.

C. Test for the source side reflections of the PKP phases:

The source-side reflections of core phases contribute more information to the study of core structures. In this study, the option to compute the seismograms, including the pPKP and



Fig. 3. P-wave velocity models near the ICB. These models were used to compute seismograms in Figure 4 and are described in more detail in the text.



Fig. 4. Synthetic seismograms of the core phases using the earth models in Figure 3. The numbers on the right of the seismograms are the relative amplitudes and are the same as those in other figures. Those seismograms with epicentral distance 159° were computed from a explosion source with a depth of 600 km.

sPKP phases, was provided as shown in Figure 5. The amplitudes of both surface reflections were dependent on the source mechanism. In some cases, the amplitude of the pPKP or sPKP phases could have been greater than that of the PKP phase for suitable radiation patterns. Hence, the stable waveforms of the latter phases yielded the same information on the primary phases of an explosion or deep events for studying the core structure. However, these later phases were not analyzed in terms of the core structure by previous studies because further implementation was required to synthesize the waveforms. The computed dense-seismic profile based on the PREM model for the core phases which included the surface reflections, is shown in Figure 6. An explosion source with a depth of 250 km was assumed in this computation. These ray paths were similar in the receiver side mantle and source side lower mantle. However, as can be seen in Figure 6, the slightly different ray paths in the earth's core near the ICB produced great waveform and amplitude variations for the primary and surface reflected core phases may provide opportunities for studying the fine structures (the vertical velocity gradient and its lateral variation) of the ICB.



Fig. 5. Synthetic seismograms for individual core phases and their summation. The final synthetic seismogram is based on the source mechanism of 52° , 30° and -50° for the strike, dip and rake angles, respectively.

D. Test for the earth core Q structure:

The waveforms of seismic waves traveling inside the earth's core are sensitive to the attenuation structure of the earth's deep interior. In this study, the option to compute the depth dependent earth Q-structures was made available. One example is shown in Figure 7. Both synthetic seismograms were computed based on the elastic media or the Q-structure proposed by the PREM model. It was found that attenuation for the PKP(DF) and PKP(AB) were different. Because the inner-core is one of the major attenuation zones of the earth (Niazi and Johnson, 1992) and the outer-core is the high Q region for the P-wave, the high frequency contents of the PKP(DF) are more attenuated than those of the PKP(AB). A test of the inner-core Q structure with short-period waveform modeling is shown in Figure 8. It was found that in the short-period frequency band the seismogram had clearly changed its amplitude and phase in both the DF and AB phases. Thus, as shown in this study, the attenuation effects of the inner core could be analyzed not only by spectrum ratios (Niazi and Johnson, 1992; Bhattacharyya *et al.*, 1993; Souriau and Roudil, 1995), but also by waveform modeling. Further testing found that the waveforms were sensitive to the depth dependent Q models.

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Fig. 6. Seismic profiles of the core phase PKP and its surface reflections (pPKP and sPKP). The characteristics of A, B, C, D and F define the branch ends of the core phases as shown in Figure 1.

E. Example for waveform model:

To demonstrate the flexibility of this GRT extension, the waveform model including the surface reflection phases was computed using the newly developed GRT code. An event from the Fiji Islands, with stable core phases, is selected. The source parameters of this Fiji Islands event have been reported by the U.S. Geological Survey (USGS) Preliminary Determination of Epicenters (PDE) as: origin time = 1996 August 5th 22hr 6 min 22.07 sec, latitude = 20.614° , longitude = 178.541° , depth = 530 km, and mb= 6.5. The best double couple solution from the USGS moment tensor solution are NP1: strike = 222° , dip = 29° , slip = -89° ; and NP2: strike = 41° , dip = 61° , slip = -91° . The above source parameters are employed in this study. The waveform model for a broadband seismogram from a GDSN station ESK (Eskdalemuir, Scotland, UK) with epicentral distance 145.1° is shown as an example of the usefulness of later



Fig. 7. Synthetic seismograms of the core phases using the PREM model. The seismogram on the top is for the elastic case and that on the bottom is for the Q model of Dziewonski and Anderson (1981).

phases for studying the earth's core structure (Figure 9a). The Green's functions are computed by the GRT using the PREM model and the source time function is selected from a P-wave seismogram of station ADK (Adak, Aleutians, Alaska) at a distance 72.2°, where the direct arrival is undisturbed by other phases. The later phases (pPKP and sPKP) are well-simulated, as shown in Figure 9a. The simulated seismic profile from epicentral distance 140° to 160°, according to the source parameters of this Fiji Islands event, is shown in Figure 9b. For the suitable source mechanism, the later phases of this event have larger amplitudes than those from an explosion source (Figure 6). Moreover, the later phases near 150° have amplitudes larger than the direct PKP phases. Of course, modeling from a single seismogram does not provide significant information about the earth's core structure. However, as demonstrated by Figure 9b, the modeling of the later phases from a dense-array with small aperture will provide significant contribution to the inner-core model of the earth.

4. DISCUSSION AND CONCLUSIONS

The waveform modeling of the earth's core structure has some limitations in the real case due to the geographic distribution of earthquakes. Deep events and nuclear explosions based on the GDSN records do not provide sufficient global coverage to study the earth's core in



Fig. 8. Comparison of the short-period seismograms from the elastic and Q models near the ICB. Both traces were generated by inserting the short-period Wood-Anderson instrument response into the synthetic seismograms of Figure 7.

detail. In previous studies, shallow events from the mid-ocean ridges and the major seismic zones were usually excluded because of the surface interaction of seismic energy in the source area. One advantage of the present study is that it provides the opportunity to study the earth's core using shallow events. Based on the methodology of this study, shallow and intermediate deep events along the major seismic zones and the mid-ocean ridges can be considered as candidates for waveform modeling. Using these events would greatly improve the global coverage of the ray paths across the earth's core. Hence, the developments tested in this study reveal the opportunity to study the earth's core in more detail.

The seismic attenuation beneath the ICB is recognized as being very complex (Niazi and Johnson, 1992; Bhattacharyya *et al.*, 1993; Souriau and Roudil, 1995), though the details of the Q-structure are not known. The attenuation structure of this region is important for understanding the evolution and dynamics of the earth's core. The mushy zone induced by innercore solidification may show lateral heterogeneity with a scale of a few thousand kilometers (Loper and Roberts, 1981). The presented depth dependent variations from different areas can be resolved by waveform modeling of the recorded seismograms in different regions of the world. The data recorded by Taiwan Seismic Network (TSN) have sampled large areas of the ICB beneath the Pacific Ocean. Besides this, the Broadband Array in Taiwan for Seismology (BATS, Kao *et al.*, 1994) also records high quality waveform data for studying the detailed structure of the ICB. Using both the spatial-dense sampled short-period travel times and the



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150°

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124

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high quality broadband waveforms, the ICB structure in a spot can be easily resolved. The proposed GRT can be used to model the waveform to infer the ICB structure beneath the Pacific Ocean based on TSN data. Furthermore, other regional seismic arrays may play important roles in the study of the ICB structure. For example, the GRSN has recorded high quality core phases from the Fiji Island deep events, while the Warramunga seismic array in northern Australia has recorded core phase seismograms from earthquake sources in central America (Wang *et al.*, 1985). Both sets of data have provided two patches of the dense-sample of the ICB, which are different from the TSN data. Finally, the core phases from the GDSN can provide the necessary global coverage for this purpose when shallow and intermediate deep events are considered. From this point of view, with the available seismic data from local, regional and global networks, the global picture of the ICB structure can be effectively enhanced.

The program developed in this study is a general purpose tool to study the earth's core. Both traveltime and waveform can be modeled using the developed GRT code. The method developed will be used to analyze real data to determine the lateral variation of the earth's core in the near future. In fact, many core phases from southern America events have been received by the TSN. The recorded array data provide unique opportunities to study in detail both the ICB and the inner-core structure under the Pacific-Ocean. One example (Huang, 1996b) is the verification of the ICB structure beneath offshore North America deviating from the standard earth models from the PREM and IASP91. The next step is to attempt to determine the lateral variations of the ICB structure and its depth-dependent anelastic properties.

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