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NOTES AND CORRESPONDENCE

Observation of Shear-wave in Lamb's Problem

Young-Fo Chang[1,*] and Chieh-Yu Liang[1]

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ABSTRACT

Lamb (1904) proposed a numerical solution of transient waves for a point source and a point receiver located on the surface of elastic homogeneous half-space. After Lamb proposed his solution, some acoustical experimentalists tried to observe these transient waves. Longitudinal-, and Rayleigh-waves propagation along surfaces were measured, but the shear-wave (S-wave) has never previously been observed until now.

In this study, small size transducers, and a high power, high performance pulse/receiver were used to excite and receive ultrasound propagation along the surface of a duralumin block with a 16-bit digital oscilloscope utilized to survey the signal. We observed the front part of an S-wave propagating with S-wave velocity along the surface, although the full waveform of the S-wave was not measured.

(Key words: Lamb's problem, Shear-wave, Ultrasound)

1. INTRODUCTION

In 1904, Lamb proposed a numerical solution for transient elastic waves for a point source and a point receiver deployed on the surface of elastic homogeneous half-space. This is now known as Lamb's problem. For long travel distances, displacement of horizontal and vertical components exhibits small amplitude for the initial arrival of longitudinal-waves (P-wave); these are followed by shear-waves (S-wave), and finally large-amplitude Rayleigh-waves (R-wave).

Many acoustical experimentalists have tried to observe these waves in laboratory settings

[1] Institute of Applied Geophysics, Institute of Seismology, National Chung Cheng University, Chia-Yi, Taiwan, ROC

* *Corresponding author address:* Prof. Young-Fo Chang, Institute of Applied Geophysics, Institute of Seismology, National Chung Cheng University,Chia-Yi, Taiwan, ROC; E-mail: seichyo@eq.ccu.edu.tw

since Lamb proposed the problem; however, observation of the S-wave has eluded their best efforts. For examples, Kaufman and Roever (1951) observed transient P- and R-waves along a wax block surface from a spark source. Others observed P-, S- and R-waves propagation in disk - a two-dimensional earth model (Oliver et al. 1954); and Tatel (1954) used two small size piezoelectric crystals as the point source and point receiver respectively, to generate and receive P- and R-waves propagation along a large steel block surface. In addition, the R-wave was induced by the impact of a small steel ball on a large steel block and detected by a small transducer (Goodier et al. 1959). Gupta and Kisslinger (1964) also generated the R-wave by an explosion in half-space (two dimensional model) and detected it using a capacitor probe. It has been noted that R-wave motion changes with depth in a Plexiglas sheet (Sorge 1965). Further similar observations were made when an explosion at the top edge of a resin sheet and a dynamic photoelasticity technique were utilized to observe elastic wave propagation in a half-space model (Dally and Thau 1967; Thau and Dally 1969). O'Brien and Symes (1971) obtained the same result in a two dimensional Perspex-aluminum model. In more recent times with the advent of more modern measurement techniques such as the Laser-ultrasound method, a non-contact method, S-wave has still proved elusive. The reason being that the low sensitivity of the receiver and its weak dynamic range (Martin et al. 1994) cannot distinguish the S-wave from R-wave on the top surface of the aluminum sample (Scales and Malcolm 2003).

Since S-waves have not been observed in Lamb's problem until now, the importance of its observation has not diminished. In this study, a less attenuated material for ultrasound is used in measuring the S-wave. As the velocity difference between the S- and R-waves is usually small in less attenuated material, a long propagation distance for the waves is necessary to help separate them. Here, the amplitude of the R-wave is greater than that of the S-wave after such a long propagation distance. Recording of such waves requires a high dynamical range apparatus. Consequently, a high power, high performance pulse/receiver, 16-bit digital oscilloscope, a large block of duralumin, and linear increment of the source-receiver distance have been adopted to observe the S-wave in Lamb's problem.

2. EXPERIMENTS AND RESULTS

Two small size P-wave transducers of 5 MHz and 3 mm in diameter (Panametrics V1091) are used as the point source and point receiver to detect vertical motion of the waves. An S-wave transducer of the same size and same frequency (Panametrics V157) detects the horizontal motion of the waves. A high power pulser/receiver (Panametrics 5058PR) is used to excite the transducer and to amplify and filter the signals detected by the transducer. A digital oscilloscope (Tektronix TDS420) surveys and digitizes the signal output from the pulser/receiver. In order to suppress random noise, each signal is averaged 1000 times in the oscilloscope. A personal computer reads the digital signal from the oscilloscope by IEEE 488 interface and stores them. Figure 1 shows a block diagram of the laboratory set-up.

A duralumin block of 14.6 cm × 14.6 cm × 11.2 cm simulates the elastic homogeneous half-space. Treacle is used as the couplant for proper contact between the transducer and duralumin. The treacle is very thin compared with the wavelength, thus its effect on the wave

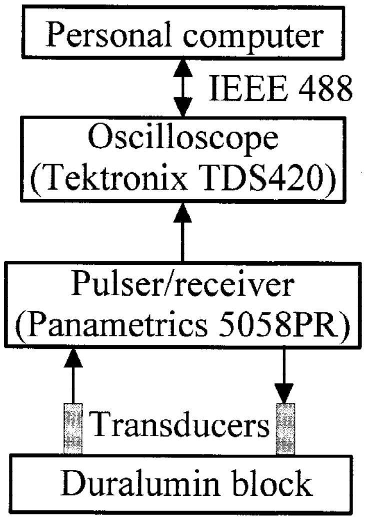


Fig. 1. Block diagram of laboratory set-up.

can be ignored (Blitz and Simpson 1996). The source and receiver are deployed at the top surface of the block. The distance between the source and receivers is from 8 to 12 cm with an increment of 0.25 cm. The P- and S-waves velocities of the duralumin are 6385 and 3155 m s^{-1}, respectively, according to the transmission measurement. The experimental measurement error in velocity is less than 0.5%. Based on Rayleigh's equation (Udias 1999), the R-wave velocity is calculated as 2945 m s^{-1}. The R-wave velocity is measured as 2941 m s^{-1}, which is consistent with the calculation. The velocity ratio of R- to S-waves is 0.93.

In order to record different amplitude levels of the waves, different driving voltages and amplifier gains are applied; however, these are fixed during measurement in profile. The horizontal component recorded by the S-wave transducer is demonstrated in Fig. 2 and the normalized gain is 10 for clear observation of the direct P-waves. The line 'Sp' in the figure represents the theoretical arrival time of the P-wave from 8 to 12 cm propagation along the surface; lines 'Ss' and 'Sr1' express the theoretical arrival times for S- and R-wave propagation along the surface respectively. The scattering phases, after the arrival of 'Sp' phase, are the "P coda waves" which are the waves scattered by the microstructure of the duralumin block. P-wave propagation along the surface was observed but we did not observe S-waves. The line 'Sr2' is the arrival time of the R-wave from 8 to 12 cm along the surface; there is an advance shift of the arrival time of the R-wave compared to the theoretical one. The dashed lines 'Br1' and 'Br2' are used to point side-boundary reflection events. The 'Br1' are events reflected from the top-corner of the model and 'Br2' are reflected from the top-edge of the model as shown in Fig. 3a. In Fig. 3b, the events 'Bw1' and 'Bw2' are the back wall echoes reflected from the bottom and bottom-corner of the model respectively.

A low normalized gain (× 1) is used to measure the R-wave, and the vertical component

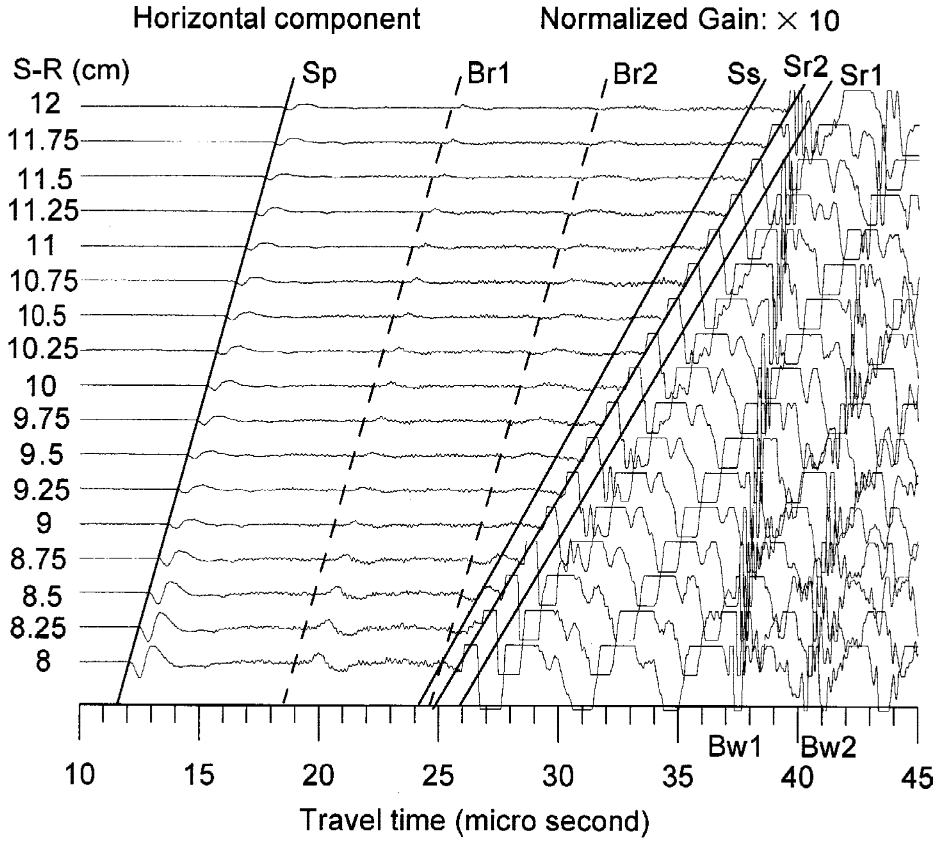
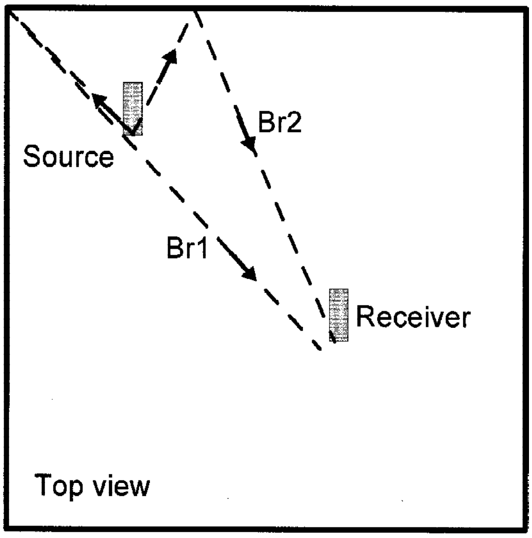


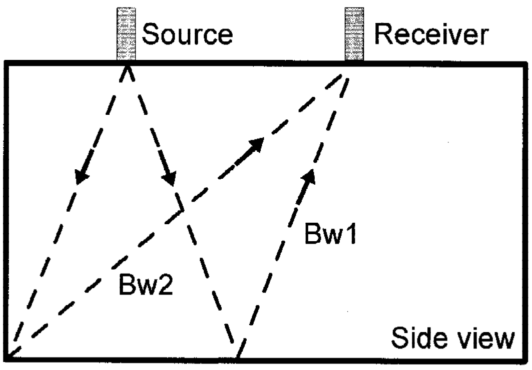
Fig. 2. Horizontal component with the reception distances 8 - 12 cm. The normalized gain is 10. The lines 'Sp', 'Ss' and 'Sr1' are theoretical arrival times for the P-, S-, and R-waves propagation along the surface, respectively. The line 'Sr2' is the arrival time of R-wave propagation along the surface. The dashed lines 'Br1' and 'Br2' are the side-boundary reflection events. The events 'Bw1' and 'Bw2' are back wall echoes reflected from the bottom of the model.

is shown in Fig. 4. Back wall events 'Bw1' and 'Bw2' are very clear. The R-wave is observed from 8 to 10.75 cm; however, it is contaminated by the 'Bw1' event from 10.75 to 12 cm. There are no significant events that could be found before the R-wave. The particle motion of the 'Sr1' and 'Sr2' phases measured at a reception distance of 6 cm for the travel time from 19.74 to 22.84 microsecond is shown in Fig. 5. This figure also shows that the phases are the R-wave.

In an attempt to observe S-wave propagation, high driving voltage and amplifier gain



(a)



(b)

Fig. 3. Ray paths of the noises. (a) Top view used to describe the ray paths of side-boundary reflection events 'Br1' and 'Br2'; (b) side view to show the ray paths of back wall reflection events 'Bw1' and 'Bw2'.

were utilized. The high-normalized gain (×100) vertical component is shown in Fig. 6. It is not just a different display gain to Fig. 4 as a four times higher driving voltage and 25 times higher amplifier gain were utilized. In Fig. 6, the P- and R-waves and 'Bw1' and 'Bw2' events are over the range of the recording system. However, the 'Br1' and 'Br2' events are clearer than those in Figs. 2, 4. The S-waves are still obscured; they are contaminated by the large amplitude R-waves keeping in step with S-waves, the P Coda waves, and the 'Br1' and 'Br2' events. Nevertheless, careful examination of Fig. 6, reveals the onset of S-wave propagation with S-wave velocity along the surface being observed from 9.75 to 12 cm; however, from 8 to 9.5 cm they are contaminated by the ‘Br2’ events.

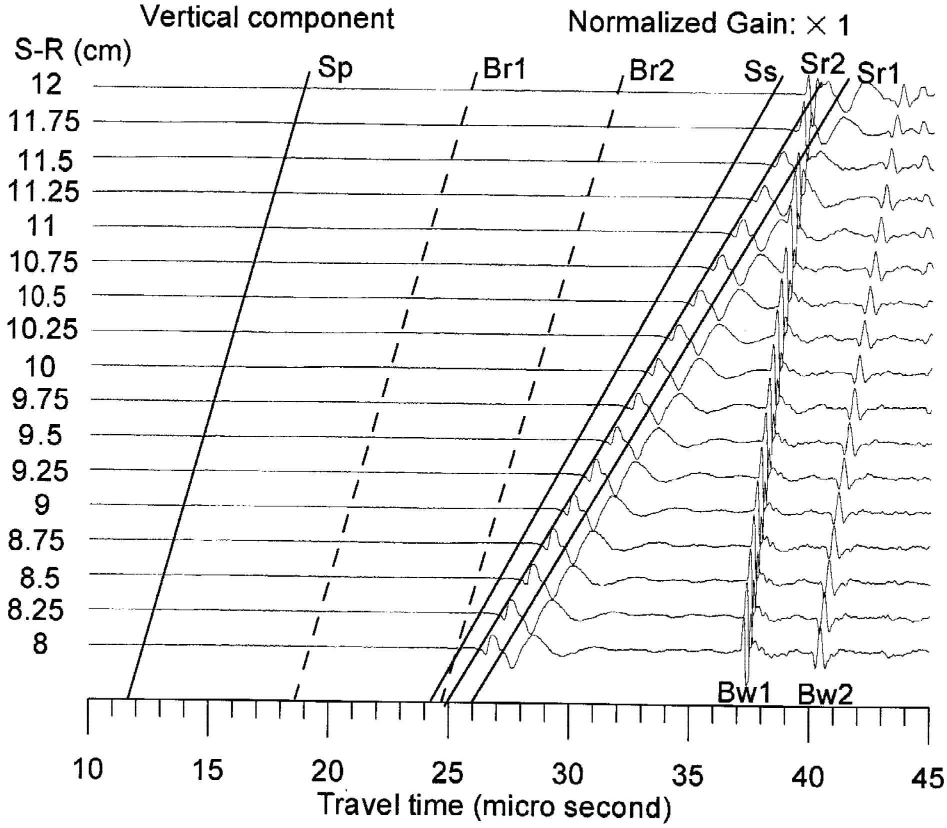


Fig. 4. The vertical component and the normalized gain is 1.

3. DISCUSSIONS AND CONCLUSIONS

S-waves radiating from a point source shot on the surface have been previously observed at the bottom of the model with an angle of about 45 degrees to the surface (Kaufman and Roever 1951; Hsieh and Chang 1996). However, the point of this paper is to verify the results proposed by Lamb in 1904. His results indicated that the surface receiver could record S-waves.

The first arrival times of direct P-waves fit very well with the theoretical arrival times of P-wave propagation along a surface (Fig. 2). Although the velocity of the R-wave is the same as the theoretical prediction, the first arrival time is 1 microsecond in advance of the theoretical arrival time of the R-wave propagation along the surface (Figs. 2, 4). The distance traveled

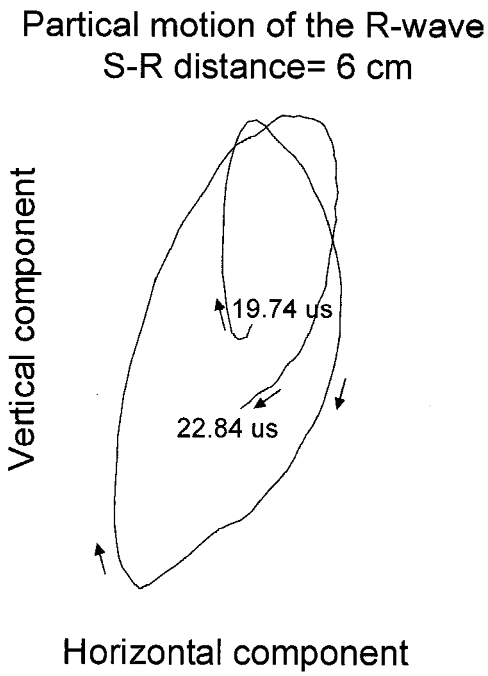


Fig. 5. Particle motion of the 'Sr1' and 'Sr2' phases.

by an R-wave propagating for 1 microsecond is 3 mm, which is just the diameter of the transducer. Therefore the advanced arrival time of the R-wave may be due to the source and receiver not being points.

The absolute amplitudes of the waves cannot be directly read from the traces because the responses of the apparatus, transducer and coupling effect are difficult to estimate. Nevertheless, amplitudinal characteristics are found in the figures. The amplitude of R-waves is greater than that of the P- and S-waves (Figs. 2, 4 and 6). The decrease in amplitude of the P-wave over the distance 8 to 12 cm is greater than that of the R-wave (Figs. 2, 4), which relates to the geometric spreading of P-wave energy being a three dimensional spherical divergence and R-wave energy being a two dimensional circular divergence.

Although 5 MHz transducers are used in the experiments, the dominant frequencies of the P-wave, R-wave and 'Bw1' event are 1, 0.5 and 2.5 MHz. No significant dispersions of the P- (Fig. 2) and R-waves (Fig. 4) are observed. The dominant wavelengths of the P- and R-waves are 0.64 and 0.59 cm respectively, and the reception distances (8 - 12 cm) are much greater than the wavelengths. The diameter of the transducer is about a half of the wavelengths of P- and R-waves. The shift of the dominant frequency of the 'Bw1' event from a high to a low frequency is caused by such a long propagation distance (greater than 23.8 cm) and high frequency energy attenuating more readily than low frequency energy. For the P- and R-waves, their wavelengths are shorter than the dominant wavelength of the wave radiated from the

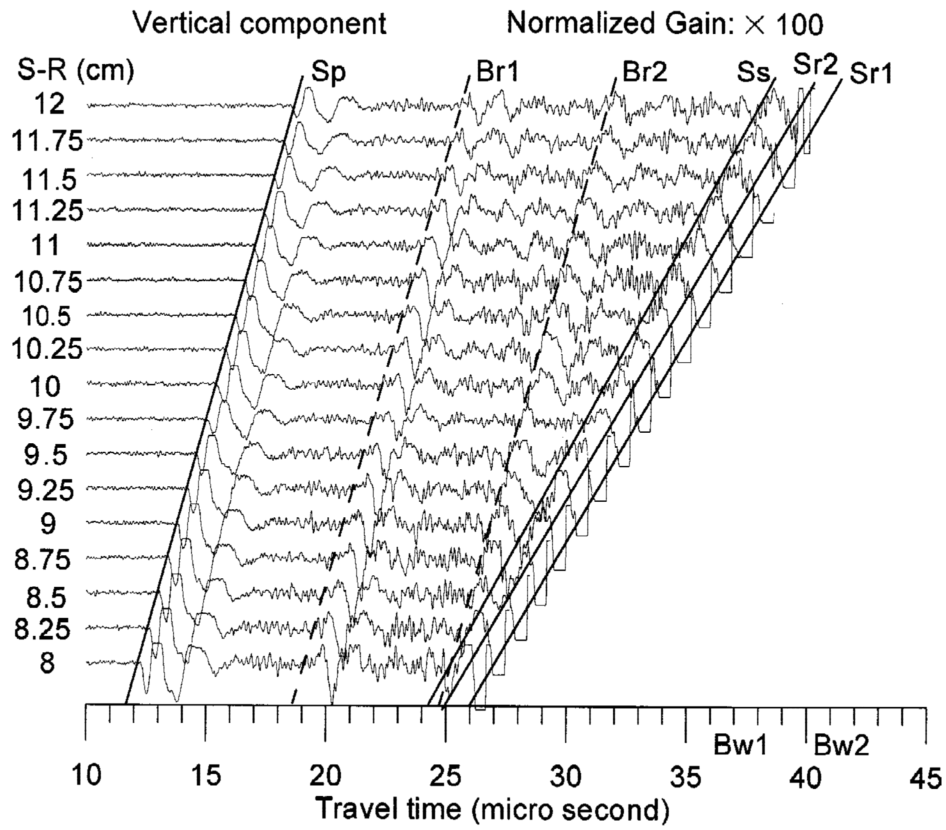


Fig. 6. The vertical component and the normalized gain is 100. The over-scaled amplitudes of the R-waves are muted in order to observe the S-waves clearly.

transducer. This phenomenon is due to the size of the receiver transducer, which is not much less then the wavelength. The receiver neutralizes the high frequency energy. Therefore their dominant frequencies are shifted to lower frequencies than those excited by the transducer.

Since the amplitude of the S-wave nearly reaches the background noise level at a 12 cm offset (Fig. 6), a further offset between the source and receiver would exceed the system's limit in recording S-waves. A larger duralumin block would be able to delay the boundary events ('Br1', 'Br2', 'Bw1' and 'Bw2') and assist in avoiding contaminating the S- and R-waves. In which case, a clean section could be expected. However, in our experiment the boundary events did not completely disturb the S- and R-waves. In any event, a larger duralumin block seems helpless for the observation of S-waves.

The power and amplification level of the apparatus are fixed during measurement in each figure, therefore the sidelobes of the transducer and the ringing of the amplifier will follow the major event and do not change with the receiver position in the figure. The slopes of lines 'Ss' and 'Sr2' are different in Fig. 6. Therefore the event propagation along the surface with S-wave velocity (line 'Ss') is not the sidelobes of the transducer or the ringing of the amplifier of the R-wave, it is the S-wave. Although the observation of a full waveform of the S-wave propagation along the surface is not clearly shown in this study, the front part of the S-wave propagation along the surface with S-wave velocity can be observed. A century after Lamb proposed his theory; we have finally been able to observe the tiny light of an S-wave propagation along a surface for a point source and a point receiver on the surface of an elastic homogeneous half-space.

REFERENCES

Blitz, J., and G. Simpson, 1996: Ultrasonic methods of nondestructive testing. Chapman & Hall, London, 22-23.

Dally, J. W., and S. A. Thau, 1967: Observations of stress wave propagation in a half-plane with boundary loading. *Int. J. Solids Struct.*, **3**, 293-307.

Goodier, J. N., W. E. Jahsman, and E. A. Ripperger, 1959: An experimental surface-wave method for recording force-time curves in elastic impacts. *J. Appl. Mech.*, **26**, 3-7.

Gupta, R. N., and C. Kisslinger, 1964: Model study of explosion-generated Rayleigh waves in a half-space. *Bull. Seism. Soc. Am.*, **54**, 475-484.

Hsieh, C. H., and Y. F. Chang, 1996: A Study of the characteristics of scattering attenuation by physical modeling. *Terr. Atmos. Ocean. Sci.*, **7**, 31-42.

Kaufman, S., and W. L. Roever, 1951: Laboratory studies of transient elastic waves. Proc Third World Petroleum Cong., Sec. I, 537-545.

Lamb, H., 1904: On the propagation of tremors over the surface on an elastic solid. *Phil. Trans. Roy. Soc.*, **A203**, 1-42.

Martin, D., B. Pouet, and P. N. J. Rasolofosaon, 1994: Laser ultrasonics applied to seismic physical modeling. In: Ebrom, D. A., and J. A. McDonald, (Eds.), Seismic physical modeling , Soc. Explor. Geophys., Tulsa, Oklahoma, p. 499-511.

Oliver, J., F. Press, and M. Ewing, 1954: Two-dimensional model seismology. *Geophysics*, **19**, 202-219.

O'Brien, P. N. S., and M. P. Symes, 1971: Model seismology. *Rep. Prog. Phys.*, **34**, 706-707.

Scales, J. A., and A. E. Malcolm, 2003: Laser characterization of ultrasonic wave propagation in random media. *Physical Review E*, **67**, 46618.

Sorge, W. A., 1965: Rayleigh-wave motion in an elastic half-space. *Geophysics*, **30**, 97-101.

Tatel, H. E., 1954: Note on the nature of a seismogram-II. *J. Geophys. Res.*, **59**, 289-294.

Thau, S. A., and J. W. Dally, 1969: Subsurface characteristics of the Rayleigh wave. *Int. J. Eng. Sci.*, **7**, 37-52.

Udias, A., 1999: Principles of seismology. Cambridge Univ. Press, London, 185p.