

# The Solar Wind Control of the Magnetopause Shape : A Comparison of A Model Magnetopause and Empirical Models

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## ABSTRACT

The solar wind control of the magnetopause shape is studied with a model magnetopause that results from three sources of magnetic field. One source is the geomagnetic field produced by the Earth's dipole. Another results from an image dipole placed in front of the magnetopause to produce the effect of the magnetopause current which limits the spatial extent of the geomagnetic field. The third is a southward interplanetary magnetic field (IMF) in the Earth's vicinity. With an image dipole strength and the location to account for the effect of the solar wind dynamic pressure ( $D_p$ ), the shape of the model magnetopause can be regarded as controlled by  $D_p$  and the southward IMF. As an image dipole strength is about fourteen times the Earth's dipole and the location is at thirty times the Earth's radius, the shape of the model magnetopause is consistent with observational results for a southward IMF. A comparison of the shape of the model magnetopause and an empirical model by Shue *et al.* (1997) shows that the image dipole strength at a fixed location correlates linearly with  $D_p$  for a southward IMF. In addition, they agree qualitatively for high  $D_p$  and a southward IMF.

(Key words: Solar wind, Magnetopause, IMF)

## 1. INTRODUCTION

The Earth's magnetosphere has generally been thought to be determined by the solar wind, the interplanetary magnetic field (IMF), and the geomagnetic field. Magnetospheric configuration is widely considered a useful means to describe the global picture of the Earth's magnetosphere. It is also conducive to understanding physical mechanisms of the formation of the Earth's magnetosphere (e.g. Nishida, 1978). In the past, there were two main models of magnetospheric configuration to describe the Earth's magnetosphere. From the balance between the dynamic pressure of the impinging solar wind and the magnetic pressure of the geomagnetic field, Chapman and Ferraro (1931) suggested that the magnetosphere might be

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closed. In order to study the cause of magnetospheric convection, Dungey (1961) proposed an open model of the magnetosphere in which the Earth's dipole is permeated by a southward IMF. Based on these two models of the magnetosphere, many studies have been conducted to explain various local magnetospheric phenomena.

In recent years, the spacecraft and satellites launched to probe the Earth's environment have collected valuable data from which some features of the magnetosphere are revealed. High-latitude HEOS 2 measurements near the dayside magnetopause showed that the magnetic field converges toward a cusp which presumably results from Chapman-Ferraro currents (Fairfield, 1977; Haerendel *et al.*, 1978). On the night side, geomagnetic field lines are interconnected with the interplanetary space (Fairfield, 1987 and references therein). The observational evidence indicates that both the convergence of the magnetic field toward the cusp and the penetration of the IMF may take place jointly. In other words, the magnetosphere could be partially closed and partially open. Thus, it is useful to construct a theoretical model to exhibit some of these features of the magnetosphere.

The terrestrial magnetopause is generally identified as the surface of separation between the geomagnetic field and the IMF. It has long been known that the shape of the magnetopause is mainly controlled by the solar wind dynamic pressure  $D_p$  and the IMF  $B_z$  component (see review by Fairfield, 1995). Recently, with the availability of large numbers of magnetopause crossings, the magnetopause shape has been fitted as a function of  $D_p$  and the IMF  $B_z$  component (Sibeck *et al.*, 1991; Petrinec *et al.*, 1991; Roelof and Sibeck, 1993; Petrinec and Russell, 1993; Shue *et al.*, 1997). Due to the limit of the functional form or satellite orbits, some empirical models are appropriate for the dayside region (e.g. Roelof and Sibeck, 1993) and some for the nightside region (e.g. Petrinec and Russell, 1993). Nevertheless, a common trend in these studies exists whereby increased pressure compresses the magnetopause earthward while IMF  $B_z$  remains steady. Moreover, the flank expands with increasing IMF  $B_z$  and the subsolar distance reduces conspicuously at low  $D_p$  and less at high  $D_p$ . Thus, comparison of a model magnetopause with empirical models helps in realizing how the magnetopause shape is controlled by the solar wind.

Recently, Yeh (1997) proposed a mathematical model to study magnetospheric structure with a southward IMF. In the model, the IMF is assumed to be due south and moderate as its magnitude is less than the geomagnetic field at the subsolar point. The model magnetosphere possesses features of both a closed model and an open model, as revealed by observations. Since the realistic magnetopause shape is more affected by IMF  $B_z$  than other components such as  $B_x$  and  $B_y$ , it seems appropriate that a model magnetopause for a southward IMF can be used to study how the magnetopause shape is controlled by the solar wind. Hence, the main goal of this study is to examine qualitatively the solar wind control of the magnetopause shape by comparing a model magnetopause based on Yeh (1997) with empirical models. This paper is organized as follows. Section 2 mathematically shows that a model magnetopause is constructed on three magnetic fields. Section 3 discusses the shape of the model magnetopause and compare it with empirical models. Finally, the last section discusses other factors affecting the comparison of the model magnetopause and empirical models, and summarizes this study.

## 2. A MODEL MAGNETOPAUSE FOR THE SOUTHWARD IMF

Referring to Yeh (1997), a mathematical model of the magnetopause shape results from three sources of magnetic field. One source is the geomagnetic field produced by the Earth's dipole. The second results from an image dipole placed in front of the magnetopause to have the effect of the magnetopause current which limits the spatial extent of the geomagnetic field. The third is a southward IMF in the Earth's vicinity. To simplify the mathematical analysis, there is no plasma in the model. Moreover, the model does not include magnetic fields induced by the field-aligned current and the magnetotail current which are important elements of the magnetosphere.

We describe the geomagnetic field by

$$\mathbf{B}_G = B_E a^3 \frac{-3xz\hat{x} - 3yz\hat{y} + (x^2 + y^2 - 2z^2)\hat{z}}{(x^2 + y^2 + z^2)^{5/2}} \quad (1)$$

in terms of a southward magnetic dipole. Its magnetic moment is equal to the equatorial field strength  $B_E$  times the cube of the Earth's radius  $a$ . The magnetic field resulting from an image dipole is described by

$$\mathbf{B}_P = B_M a^3 \frac{-3(x - r_M)z\hat{x} - 3yz\hat{y} + ((x - r_M)^2 + y^2 - 2z^2)\hat{z}}{((x - r_M)^2 + y^2 + z^2)^{5/2}} \quad (2)$$

in terms of a southward dipole placed at  $x = r_M$  on the  $x$ -axis. This image dipole in front of the magnetopause has a magnetic moment  $B_M a^3$  greater than the geomagnetic moment. Namely,  $B_M > B_E$  and  $x_M > a$ . The IMF is described by

$$\mathbf{B}_I = B_I \hat{z} \quad (3)$$

in terms of the strength of the IMF  $B_I$ , and its direction points southward for  $B_I < 0$  and northward for  $B_I > 0$ . Here we use the Cartesian coordinates with the origin located at the Earth's center, the  $x$ -axis pointing toward the Sun, the  $y$ -axis toward the dusk, and the  $z$ -axis toward geomagnetic north. The total magnetic field is thus given by

$$\mathbf{B}_G + \mathbf{B}_P + \mathbf{B}_I = B_x \hat{x} + B_y \hat{y} + B_z \hat{z} \quad (4)$$

with

$$B_x = B_E a^3 \frac{-3xz}{(x^2 + y^2 + z^2)^{5/2}} + B_M a^3 \frac{-3(x - r_M)z}{((x - r_M)^2 + y^2 + z^2)^{5/2}} \quad (5)$$

$$B_y = B_E a^3 \frac{-3yz}{(x^2 + y^2 + z^2)^{5/2}} + B_M a^3 \frac{-3yz}{((x - r_M)^2 + y^2 + z^2)^{5/2}} \quad (6)$$

$$B_z = B_E a^3 \frac{x^2 + y^2 - 2z^2}{(x^2 + y^2 + z^2)^{5/2}} + B_M a^3 \frac{(x - r_M)^2 + y^2 - 2z^2}{((x - r_M)^2 + y^2 + z^2)^{5/2}} + B_I \quad (7)$$

Since regular field lines are smooth curves, these can provide a vivid visualization of the magnetopause shape. Regular field lines emanate from and terminate at neutral points at which the magnetic field strength becomes zero. The model magnetopause is a flux surface in which regular field lines emanate from and terminate at cusps corresponding to neutral points. Hence, the location of cusp neutral points plays a predominant role in the magnetopause shape. Before proceeding to draw magnetic field lines, it is essential to find out the location of cusp neutral points in the model.

In the model, neutral points occur where  $B_G + B_P$  is canceled out by  $B_I$ . For the  $y = 0$  plane on which a cusp neutral point is located, by combining (5) and (7), the location of the cusp neutral point is determined by

$$\frac{B_M}{B_E} = \frac{x((x - r_M)^2 + z^2)^{5/2}}{(r_M - x)(x^2 + z^2)^{5/2}} \quad (8)$$

and

$$\frac{B_I}{B_E} = \frac{-r_M(r_M x - x^2 - 2z^2)a^3}{(r_M - x)(x^2 + z^2)^{5/2}} \quad (9)$$

Since the model is assumed to be permeated by a southward IMF, the magnetopause shape appears symmetrical in both the  $y = 0$  and  $z = 0$  planes. Accordingly, the location of north and south cusp neutral points are symmetrical in the  $z = 0$  plane. In this study, the distance from the Earth is normalized by the Earth's radius  $a$  and an image dipole strength is normalized by the magnetic field strength  $B_E$  at the Earth's equator. However,  $B_I$  is converted to SI units by multiplying  $B_I/B_E$  by  $B_E = 3.12 \times 10^4 \text{ nT}$ . Instead of directly solving (8) and

(9) for the location of cusp neutral points, we superimpose the contours of various  $B_M/B_E$  and  $B_I$  values with arbitrary  $r_M$  in the  $x-z$  plane and find out the location of north and south cusp neutral points using the intersection of both contours. In situ measurements (e.g. Roelof and Sibeck, 1993) have shown that on average, the subsolar distance is  $10a$ . The subsolar distance is the standoff distance of the subsolar point where the solar wind dynamic pressure is balanced by the Earth's magnetic pressure. Hence, it is appropriate to choose a normalized  $r_M$  roughly twice the subsolar distance. For example,  $r_M$  is given as 20 (see Figure 1) and contours of various values of  $B_M/B_E$  and  $B_I$  are plotted over  $4 \leq x \leq 9$  and  $4 \leq z \leq 9$  in the meridian plane. Note that dotted lines denote  $B_M/B_E$  and solid lines  $B_I$ . With  $r_M=20$ ,  $B_M/B_E=8.6$  and  $B_I = -5nT$ , Figure 1 shows that the north cusp neutral point is located at ( $x=5.7$ ,  $z=6.2$ ). This cusp location is consistent with satellite observations (cf. Frank, 1971). As a result, the location of cusp neutral points in the model is determined using  $r_M$ ,  $B_M/B_E$  and  $B_I$ .

For given values of  $r_M$ ,  $B_M/B_E$  and  $B_I$ , the flux surface of the magnetopause is constructed on the spatial extent of coplanar field lines leaving/entering north/south cusp neutral points. The integration of field lines is described by a differential equation as follows

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z} = \frac{ds}{B} \quad (10)$$

where  $(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2$  and  $B^2 = B_x^2 + B_y^2 + B_z^2$ . Magnetic field lines in the neighborhood of a neutral point are determined by a linearized equation which can be derived from (10) with Taylor's expansion at the neutral point. Thus, the shape of the model magnetopause can be formed with footpoints at which coplanar field lines emanating from the south cusp neutral point are intercepted by the equatorial plane. The subsolar distance in the model is the location of one of the coplanar field lines emanating from the south cusp neutral point intercepted by the  $x$ -axis. Since the magnetic field strength reaches a minimum between the Earth's dipole and an image dipole, the subsolar distance in the model is limited by  $x_M = r_M / (1 + (B_M/B_E)^{1/4})$  where  $\partial B_z / \partial x = 0$ . The flank distance in the model is the location of one of the coplanar field lines emanating from the south cusp neutral point intercepted by the  $y$ -axis. As a result, the subsolar distance and the flank distance in the model are determined by a southward IMF, an image dipole strength and the location. In other words, the shape of the model magnetopause is determined by  $r_M$ ,  $B_M/B_E$  and  $B_I$ .

### 3. THE MODEL MAGNETOPAUSE AND EMPIRICAL MODELS

As magnetohydrodynamic interaction of the terrestrial magnetosphere and flowing solar wind is in a state of equilibrium, the magnetopause current will be induced to shield the impinging solar wind. To balance the solar wind dynamic pressure  $D_p$ , the magnetic flux induced by the magnetopause current will enhance magnetic pressure at the dayside magnetopause. In the steady state, the magnetic pressure at the subsolar point is equal to  $D_p$ . In other words, the magnetic flux induced by the magnetopause current is proportional to  $D_p$ . With an

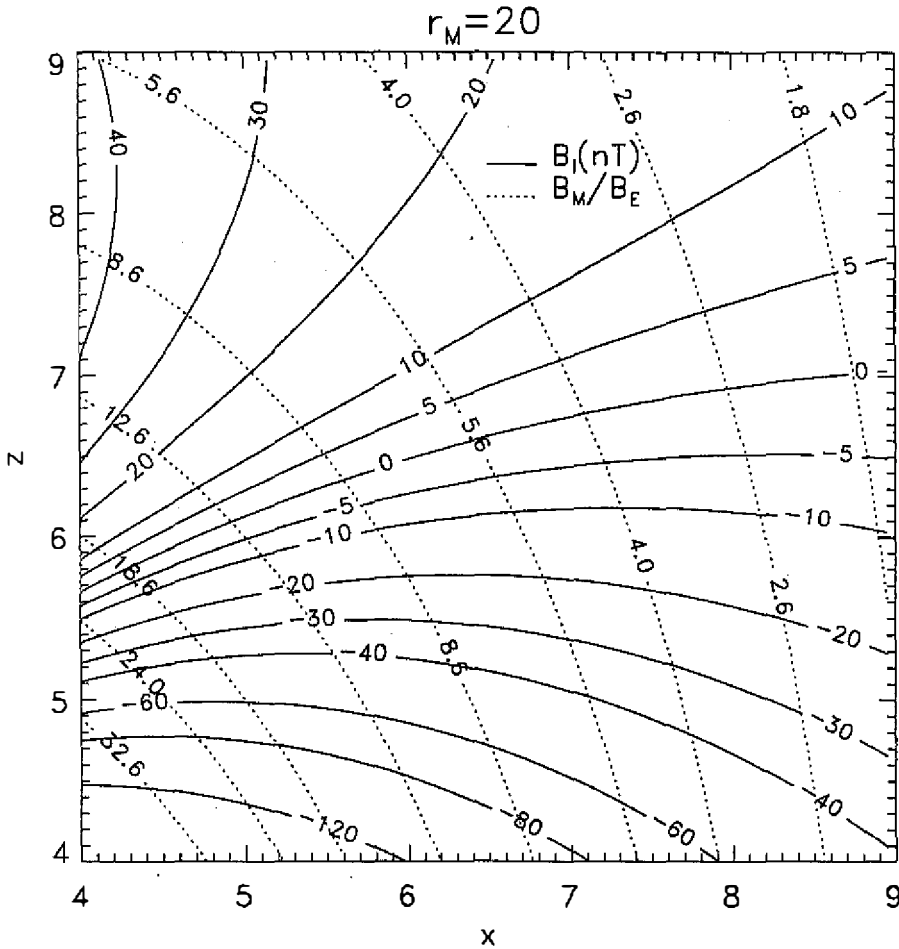


Fig. 1. An example of contours of various  $B_M/B_E$  and  $B_I$  (nT) for  $r_M=20$  superimposed in  $x - z$  plane. Dotted lines denote  $B_M/B_E$  and solid lines  $B_I$ . Cusp neutral points can be located at the intersection of both contours of  $B_M/B_E$  and  $B_I$ .

image dipole and the location to produce the magnetic flux induced by the magnetopause current, the model magnetopause is an approximation of the dayside magnetopause. Hence, the shape of the model magnetopause can be regarded as being controlled by  $D_p$  and a southward IMF. In order to investigate how the magnetopause shape is controlled by the solar wind, we attempt to compare the shape of the model magnetopause with empirical models.

With an image dipole and the location to account for the effect of  $D_p$ , we first compare the shape of the model magnetopause with an empirical model by Roelof and Sibeck (1993). An ellipse of evolution was used by Roelof and Sibeck (1993) to fit the magnetopause shape, and they studied it by dividing magnetopause crossings into each bin of dynamic pressure  $D_p$  according to IMF  $B_z$  and IMF  $B_z$  according to  $D_p$ , respectively. Figure 9 of their cor-

rected version (Roelof and Sibeck, 1994) shows that the subsolar distance is about  $8a - 12a$  and the flank distance is about  $15a - 25a$  for various values of  $D_p$  and IMF  $B_z$ . In Figure 9 of Roelof and Sibeck (1994) there is a trend whereby for a constant IMF  $B_z$ , the magnetopause moves earthward in a similar shape with increasing pressure. In addition, their results show that the magnetopause around the subsolar point moves earthward and the flank expands with increasing IMF  $B_z$  for a constant dynamic pressure.

In the second section, we mentioned that the shape of the model magnetopause is determined by  $r_M$ ,  $B_M/B_E$  and  $B_I$ . This means that the subsolar distance and the flank distance in the model are determined by  $r_M$ ,  $B_M/B_E$  and  $B_I$ . For convenience of comparison with empirical results, we have to choose adequate values of  $r_M$ ,  $B_M/B_E$  and  $B_I$  to calculate the shape of the model magnetopause. In the model, the subsolar distance is less than  $x_M = r_M / (1 + (B_M/B_E)^{1/4})$ . Hence, the subsolar distance is determined by  $r_M$  and  $B_M/B_E$ . In Figure 2, it is shown that  $x_M$  varies as  $r_M$  and  $B_M/B_E$  change. In the five

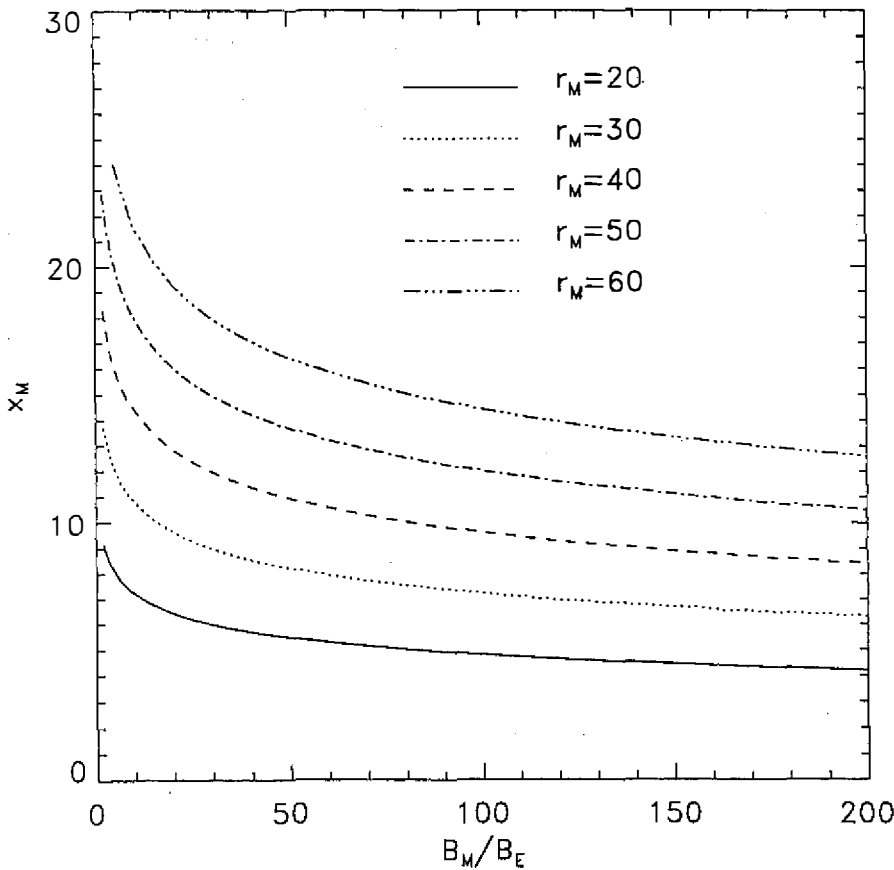


Fig. 2. A diagram of the relation between  $B_M/B_E$  and  $x_M$  for various  $r_M$ :  $r_M=20$  (solid);  $r_M=30$  (dotted);  $r_M=40$  (dashed);  $r_M=50$  (dashed-dotted);  $r_M=60$  (dashed-dotted-dotted).

curves in Figure 2, the solid curve is for  $r_M=20$ , the dotted curve for  $r_M=30$ , the dashed curve for  $r_M=40$ , the dashed-dotted curve for  $r_M=50$  and the dashed-dotted-dotted curve for  $r_M=60$ . Figure 2 shows that  $x_M$  is close to 10 when  $r_M$  has values of 30, 40, 50, 60 which correspond to  $B_M/B_E=14, 72, 190, 440$ , respectively. However, in Figure 3, it is shown that the flank distance shrinks as  $r_M$  and  $B_M/B_E$  increase. In contrast, the flank distance for  $r_M=30$  and  $B_M/B_E=14$  is consistent with the empirical fitting by Roelof and Sibeck (1994). By using this procedure, as shown in Figure 4, two curves are plotted for  $r_M=30$ ,  $B_I=-2.5nT$  and values for  $B_M/B_E=14$  and 14.5. To compare the observational result in the same unit, we

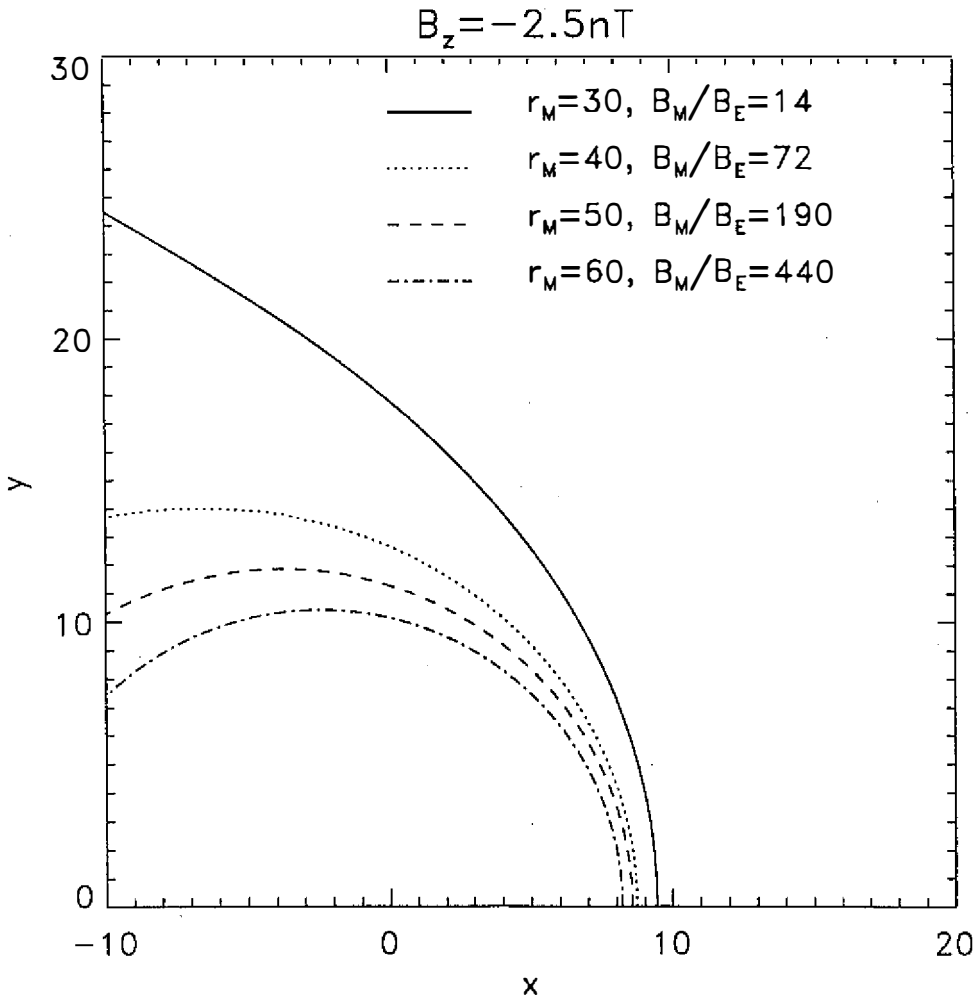


Fig. 3. The shape of the model magnetopause shape for  $B_z = -2.5nT$  and various values of  $B_M/B_E$  and  $r_M$ :  $r_M=30$  and  $B_M/B_E=14$  (solid);  $r_M=40$  and  $B_M/B_E=72$  (dotted);  $r_M=50$  and  $B_M/B_E=190$  (dashed);  $r_M=60$  and  $B_M/B_E=440$  (dashed-dotted).



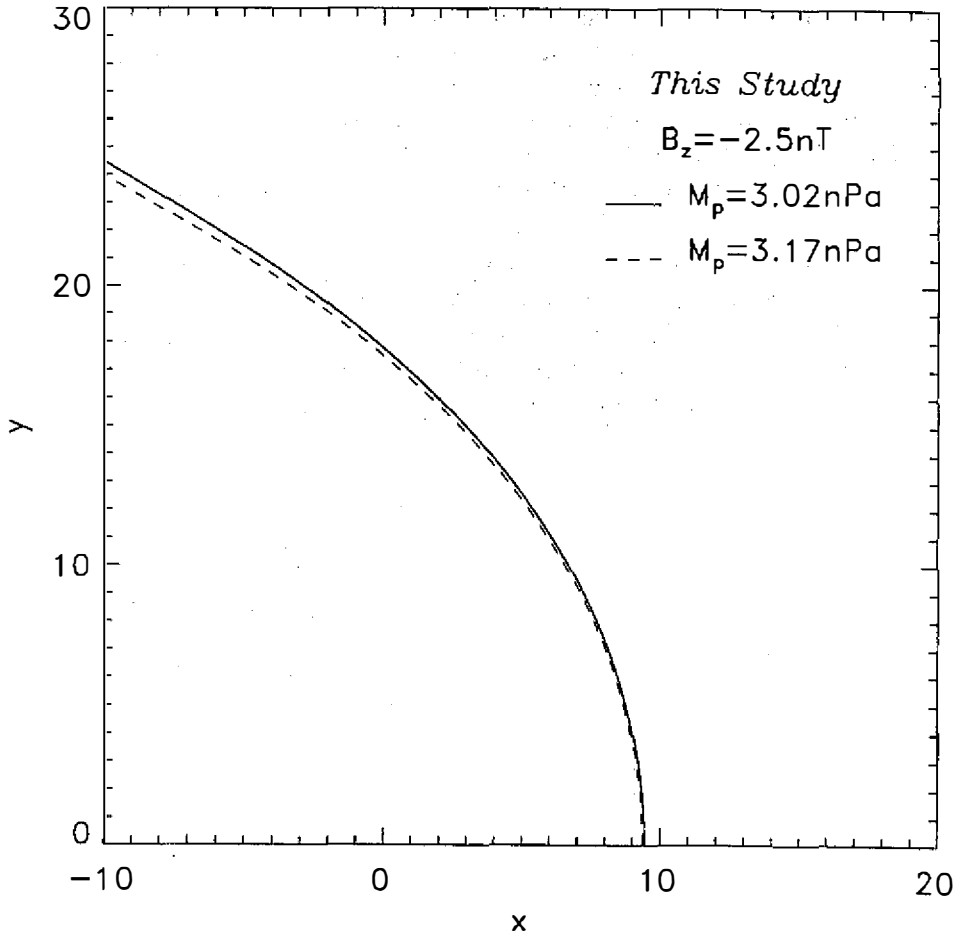


Fig. 4. The shape of the model magnetopause for  $r_M = 30$ ,  $B_z = -2.5 \text{ nT}$  and various values of  $M_P$ :  $M_P = 3.02 \text{ nPa}$  (solid);  $M_P = 3.17 \text{ nPa}$  (dashed).

approximate  $D_p$  to the magnetic pressure at the subsolar point in the model and henceforth,  $B_I$  is rewritten as  $B_z$ . In the model, the magnetic pressure at the subsolar point is expressed as

$$M_P = \frac{B_{zs}^2}{2\mu_0} \quad (11)$$

where  $B_{zs}$  is the magnetic field at the subsolar point and  $\mu_0$  is the permeability in space. Thus,  $B_M/B_E = 14$  corresponds to  $M_P = 3.02 \text{ nPa}$  and  $B_M/B_E = 14.5$  for  $M_P = 3.17 \text{ nPa}$ . In Figure 4, the solid curve is for  $M_P = 3.02 \text{ nPa}$  and the dashed curve for  $M_P = 3.17 \text{ nPa}$ .

Note that the shape of the model magnetopause moves earthward in a similar way with increasing  $M_p$  for a constant  $B_z$ . It is evident from Figure 4 that there is a qualitative trend of dependence of the magnetopause shape upon the solar wind dynamic pressure for a constant  $B_z$ . The two curves in Figure 5 are for,  $r_M=30$ ,  $M_p = 3.3nPa$  (corresponding to  $B_M/B_E=15$ ) and for  $B_z = -2.5$  (dashed) and  $-5.0nT$  (solid). In Figure 5, there is a trend of the flank expanding with increasing southward  $B_z$  for a constant  $D_p$ . However, the subsolar distance in Figure 5 seems constant for varying southward  $B_z$ . This is due to the subsolar distance in the model being limited by  $x_M = r_M / (1 + (B_M/B_E)^{1/4})$  which is independent of the southward  $B_z$ . Roelof and Sibeck(1993) also studied dependence of the subsolar distance upon  $D_p$  and southward  $B_z$ . They found that the subsolar distance varies less sensitively at high  $D_p$  for varying southward  $B_z$ . Hence, Figure 5 may be for high  $D_p$  and not for low  $D_p$ . By contrast, the shape of the model magnetopause is qualitatively consistent with the empirical fitting for a

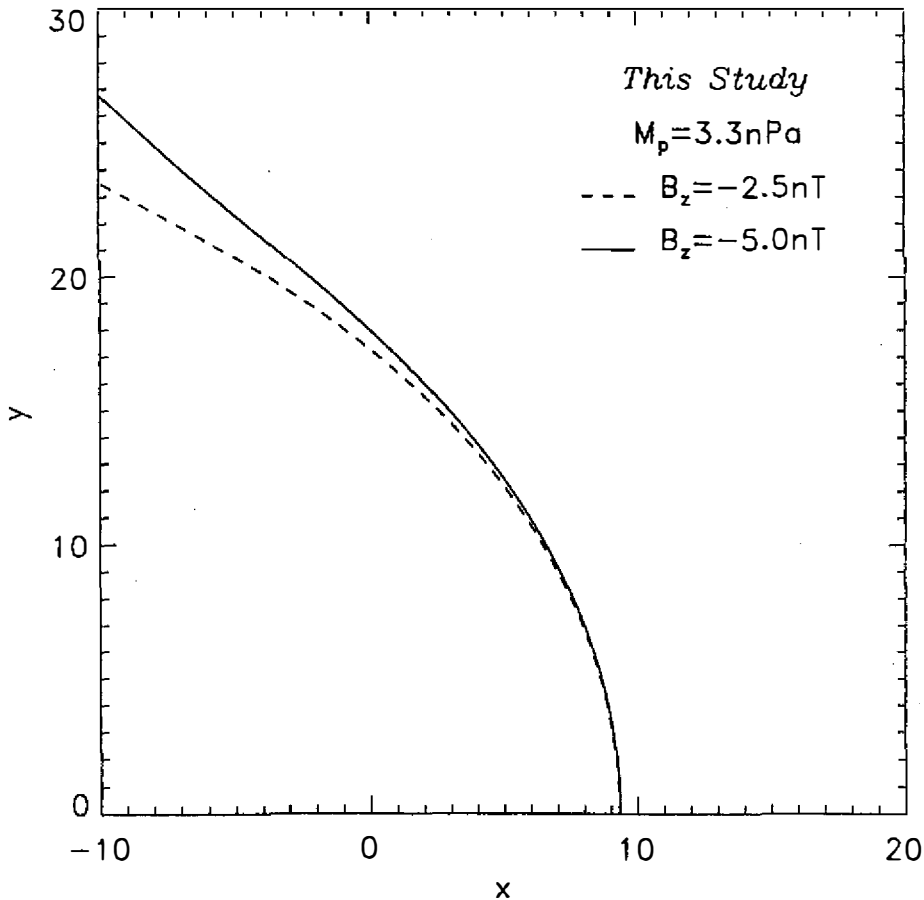


Fig. 5. The shape of the model magnetopause for  $r_M=30$ ,  $M_p = 3.3nPa$  and various values of the southward IMF:  $B_z = -5.0nT$  (solid);  $B_z = -2.5nT$  (dashed).

southward IMF. However, owing to a lack of variation of the subsolar distance in the model magnetopause, it is appropriate for high  $D_p$  only by comparison. Nevertheless, it implies that the magnetopause shape can be fitted as a function of the southward  $B_z$ , the image dipole strength and the location.

In order to verify whether an image dipole strength and the location can account for the effect of  $D_p$ , we investigate their relation with a comparison of the magnetopause shape fitted by a functional form proposed by Shue *et al.*(1997). According to Shue *et al.*(1997), the magnetopause shape can be fitted as

$$r = r_0 \left( \frac{2}{1 + \cos\theta} \right)^\alpha \quad (12)$$

where  $r$  is the radial distance from the Earth's center to the magnetopause,  $r_0$  is the distance from the Earth's center to the subsolar point and  $\alpha$  is the exponential factor related to magnetopause flaring. For a southward IMF, the subsolar distance  $r_0$  is derived as

$$r_0 = (11.2 + 0.14B_z)(D_p)^{-1/6.6} \quad (13)$$

and the flaring factor  $\alpha$  is expressed as

$$\alpha = (0.58 - 0.01B_z)(1 + 0.01D_p) \quad (14)$$

From (13),  $D_p$  can be determined for given  $r_0$  and a southward IMF  $B_z$  component. The subsolar distance in the model is the location of one of the coplanar field lines emanating from the south cusp neutral point intercepted by the  $x$ -axis. Hence, the subsolar distance  $r_0$  can be acquired and substituted into (12) with a given  $B_z$  to get the corresponding value of  $D_p$ . In this study, the subsolar distance  $r_0$  in the model is calculated with  $B_M/B_E$  varying from 14 to 16 for  $r_M=30$  and the southward IMF  $B_z = -2.5$  and  $-5.0nT$ . Figure 6 shows that  $M_p$  increases with increasing  $B_M/B_E$ . In Figure 6, the dotted line is for  $B_z = -2.5nT$  and the solid line for  $B_z = -5.0nT$ . The relation between  $D_p$  and  $B_M/B_E$  is shown in Figure 7, with a dotted line for  $B_z = -2.5nT$  and a solid line for  $B_z = -5.0nT$ . It is obvious from Figure 7 that  $D_p$  increases as  $B_M/B_E$  increases. Comparison of Figures 6 and 7 shows that for the southward IMF, the effect exerted by the solar wind dynamic pressure upon the dayside magnetopause can be approximated with an image dipole strength and the location.

To justify whether Figures 4 and 5 are for high  $D_p$  and the southward IMF, we reconstruct the magnetopause shape with the empirical model for the same parameters. By using (11), (12) and (13), the magnetopause shape based on the Shue *et al.*(1997) model is shown in Figures 8 and 9. Figure 8 shows the magnetopause shape for  $B_z = -2.5nT$  with  $D_p = 3.02nPa$  (solid) and  $D_p = 3.17nPa$  (dashed). The magnetopause shape in Figure 9 is for  $D_p = 3.3nPa$  with the southward IMF values of  $B_z = -2.5nT$  (dashed) and  $B_z = -5.0nT$

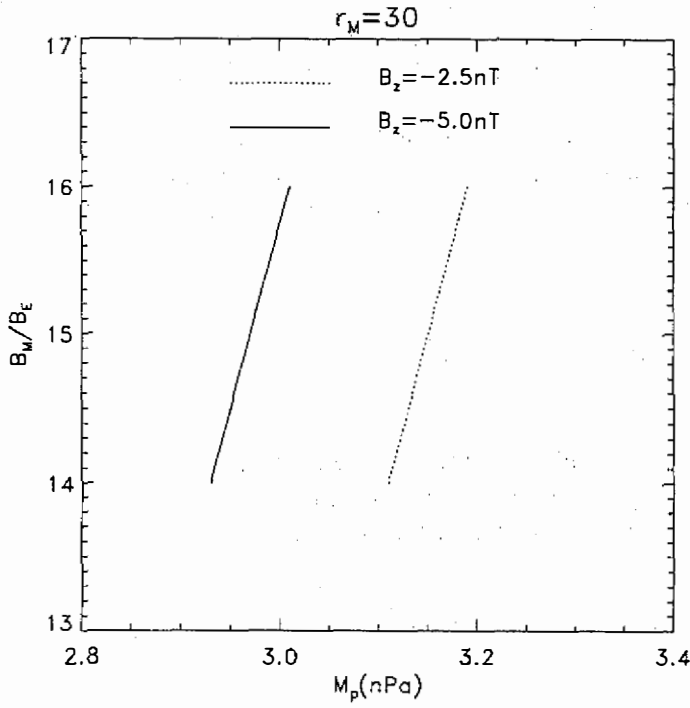
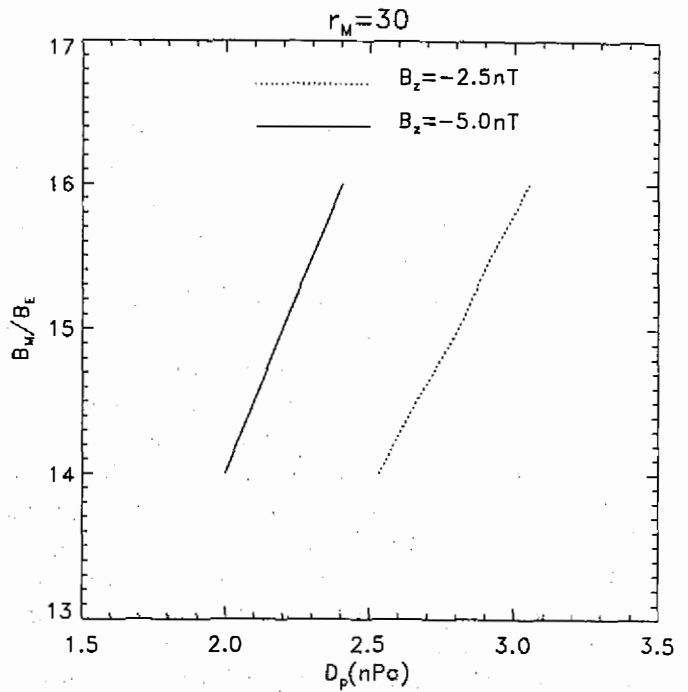


Fig. 6. A diagram of the relation between  $B_M/B_E$  and  $M_P$  for  $r_M=30$  and various values of the southward IMF:  $B_z = -5.0 \text{ nT}$  (solid);  $B_z = -2.5 \text{ nT}$  (dotted).

Fig. 7. A diagram of the relation between  $B_M/B_E$  and  $D_p$  for  $r_M=30$  and various values of the southward IMF:  $B_z = -5.0 \text{ nT}$  (solid);  $B_z = -2.5 \text{ nT}$  (dotted).



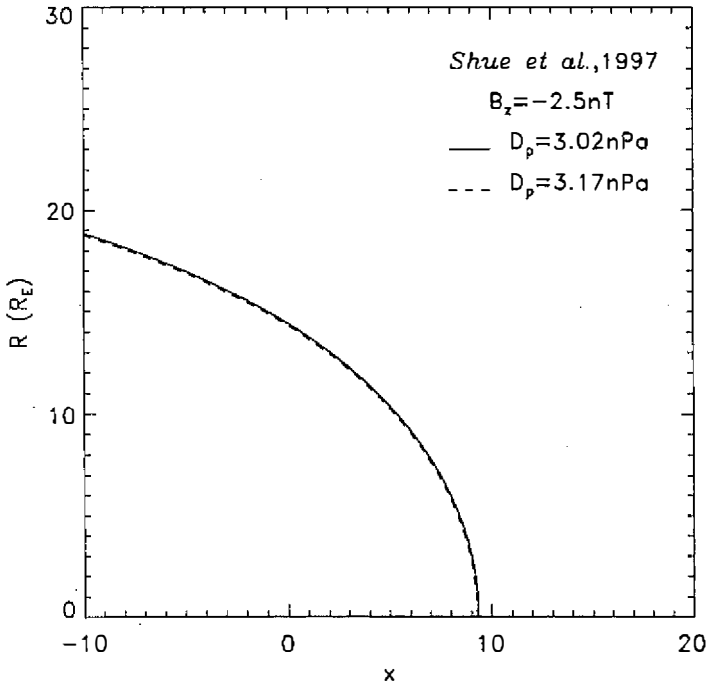
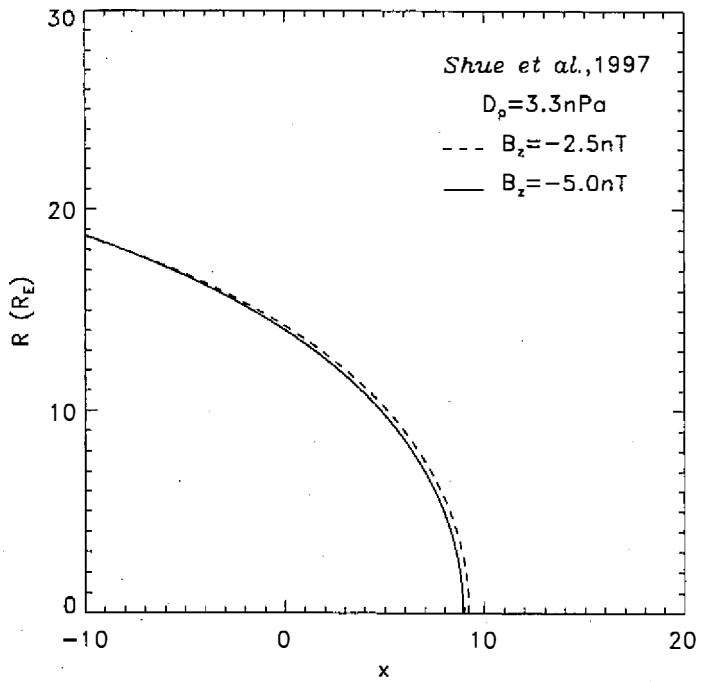


Fig. 8. The magnetopause shape for a southward IMF  $B_z = -2.5nT$  and various values of dynamic pressures reproduced from the Shue *et al.*(1997) model:  $D_p = 3.02nPa$  (solid);  $D_p = 3.17nPa$  (dashed).

Fig. 9. The magnetopause shape for  $D_p = 3.3nPa$  and various values of the southward IMF reproduced from the Shue *et al.*(1997) model:  $B_z = -5.0nT$  (solid);  $B_z = -2.5nT$  (dashed).



(solid). In contrast, for the dayside region, both models of the magnetopause shape are consistent with each other except for the flank distance. Hence, we justify that Figures 4 and 5 are for high  $D_p$  and a southward IMF.

#### 4. DISCUSSION AND SUMMARY

In this study, the shape of the model magnetopause is obtained in the equatorial plane. Since the dayside magnetopause is axis-symmetric, the shape of the model magnetopause can be regarded as an approximation of the magnetopause shape observed by satellites in low-latitude orbits. For the same subsolar distance and the same flank distance,  $M_p$  in Figures 4 and 5 seems larger than  $D_p$  in the empirical fitting by Roelof and Sibeck (1994). This implies that the geomagnetic field at the subsolar point may be overestimated with the Earth's dipole and given image dipole. On the other hand, there is a view that the reduction of the subsolar distance may result from the flux erosion due to the subsolar reconnection (e.g. Paschman *et al.*, 1986). In this study, the model magnetosphere is based on a mathematical model by Yeh (1997) which is characterized by a pair of cusp neutral points in the noon meridian and a segmental neutral line on the equatorial plane. In a steady state, magnetic reconnection in this model magnetosphere occurs at the flank with the exception of at the subsolar point. Hence, this model magnetosphere may not explain the flux erosion in the subsolar region. In Figure 5, there is also a trend whereby increased  $B_z$  enhances tail flaring as  $D_p$  corresponding to  $M_p$  remains constant. This result is consistent with the prediction by Petrinec and Russell (1993) that tail flaring is associated with a southward IMF  $B_z$ . It is worth noting that the flank distance in Figures 4 and 5 are inconsistent with Figures 8 and 9. This discrepancy shows that the dipole's strength  $B_M/B_E$  in the theoretical model may not account for the total effect of the solar wind. Nevertheless, the model provides a preliminary theoretical explanation on how the magnetopause shape is controlled by the solar wind.

Instead of solving the location of a cusp neutral point analytically, we have presented an alternative approach to finding the location by using the intersection of both contours of  $B_M/B_E$  and  $B_I$  for a specific  $r_M$  in the meridian plane. By using the integration of coplanar field lines emanating from a south cusp neutral point, we have constructed the shape of the model magnetopause which consists of the footpoints of coplanar field lines in the equatorial plane. With an image dipole strength and the location to account for the effect of the solar wind dynamic pressure, the shape of the model magnetopause is compared to the empirical model by Roelof and Sibeck (1993) for a southward IMF. With  $B_M/B_E \sim 14$  and  $r_M = 30$ , the shape of the model magnetopause is consistent with the empirical fitting by Roelof and Sibeck (1994) for a southward IMF. The comparison shows that in both models the dayside magnetopause moves earthward with increased dynamic pressure for a constant southward IMF. Moreover, the flank in both models expands as the southward IMF increases for a constant  $D_p$ . The relation between the image dipole strength at a fixed location and  $D_p$  is examined by using a functional form by Shue *et al.* (1997), and it shows that they are linearly correlated. Hence, it is verified that the effect exerted by the solar wind dynamic pressure can be approximated by an image dipole strength and the location. With the exception of the flank distance, for high  $D_p$  and southward IMF, the magnetopause shape based on Shue *et al.* (1997) is consistent with our results.

In summary, this study provides a qualitative description on the solar wind control of the magnetopause shape and it shows that the magnetopause shape can be fitted as a function of the southward IMF, the image dipole strength and the location. In addition, the shape of the model magnetopause is consistent with empirical models for high  $D_p$  with a southward IMF.

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