Magnitude Scales and Their Relations for Taiwan Earthquakes: A Review

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ABSTRACT

The magnitude scales, including $M_L$, $M_D$, $M_s(GR)$, $m_B$, $M_s$, $m_B$, $M_H$, $M_J$ and $M_L$, applied to quantify earthquakes in the Taiwan region since 1900 are reviewed. Their relations studied by several authors are also discussed.

1. INTRODUCTION

Magnitude is essentially a directly measurable parameter to quantify earthquakes. Since Richter introduced local magnitude in 1935, numerous magnitude scales have been defined and widely used for scientific and practical purpose, for examples, the study on seismicity, the estimation of seismic risk, and earthquake prediction research. The magnitude scales are defined based on different types of seismic waves at different periods of oscillation. Some magnitude scales are not used for the whole time period since 1900. It is necessary to understand the difference and relation between two magnitude scales for establishing a complete earthquake catalogue. Miyamura (1978), Bát (1981), Chung and Bernreuter (1981), and Utsu (1982b) reviewed various magnitude scales and their relations in detail.

Taiwan is a seismologically active region. Historically, a lot of destructive earthquakes shook the region and caused severe damage. Several catalogues, e.g. CMO (1952), Gutenberg and Richter (1954), Duda (1965), Rothen (1969), Hsu (1971, 1980 and 1985), Lee et al. (1978), Bát and Duda (1979), Utsu (1979 and 1982a), Abe (1981 and 1984), Abe and Kanamori (1980), Abe and Noguchi (1983a,b), Yeh and Hsu (1985) and Chen and Yeh (1989), include Taiwan earthquakes in different time intervals. A catalog including four volumes for each year has been published by the Central Weather Bureau (CWB, formerly Taiwan Weather Bureau) since 1954. During 1973-1991, a catalog including four volumes for each year was published by the Institute of Earth Sciences, Academia Sinica. Recently, the two catalogues were merged and the new catalog is published by the CWB. In these catalogs, different magnitude scales were used. The relations among the magnitude scales were studied by numerous authors, e.g. Liaw and Tsai (1981), Yeh et al. (1982), Wang (1985), Shin (1986), Wang and Chiang (1987), Cheng and Yeh (1989), Li and Chiu (1989), Wang et al. (1989, 1990), and Wang and Miyamura (1990).

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In this paper, the magnitude scales used for quantifying Taiwan earthquakes and their relations will be reviewed in detail. The materials are mainly from the papers published in numerous journals. Also included are a few current results done by the author.

2. MAGNITUDE SCALES

(1) Local Magnitude

Richter (1935) defined the local magnitude $M_L$ based on the amplitudes recorded on the Wood-Anderson torsion seismographs with natural period of 0.8 sec, damping factor of 0.8 and magnification of 2800. Richter defined the earthquake, for which the maximum trace amplitude at a distance of 100 km is 1 mm, to be the zero-magnitude earthquake. If $A_o(\Delta)$ expresses the function of the maximum trace amplitude $A_o$ of the zero-magnitude earthquake in terms of epicentral distance $\Delta$, then $M_L$ is given by:

$$M_L = \log A(\Delta) - \log A_o(\Delta)$$

where $A$ is the maximum trace amplitude on the Wood-Anderson seismograph for the earthquake at a distance $\Delta$. A table of $-\log A_o$ as a function of distance $\Delta$ (in kilometers) can be found in the text by Richter (1958). Eq. (1) was originally determined only for the southern California earthquakes and for the maximum trace amplitudes with periods of between 0.0 and 0.5 sec, for which the magnification is 2800 for the Wood-Anderson seismograph. The attenuation of seismic waves in this period range is mainly caused by the absorptive properties of the upper layer of the earth's crust. Hence, wide variation in the amplitude versus distance relations over the surface of the earth's crust must be remarkable. However, the $M_L$ scale has been widely used in other geological provinces without regional corrections.

In 1980, a Wood-Anderson seismograph with a magnification of 100, manufactured by Geotech Co., USA was operating at the Institute of Earth Sciences (IES), Academia Sinica. Unfortunately, the seismograph was out of service after 1980. Since 1980, a simulated Wood-Anderson seismograph from a L-4C sensor has been installed at the Institute (Liu, 1981; Wang et al., 1989). Liu (1981) measured the maximum amplitudes of six earthquakes recorded by the two seismographs at the same time. The ratios of the two maximum trace amplitudes change from 0.96 to 1.04 with the average of 1.0, thus indicating that the simulated one can work as a real one. Since 1980, the local magnitudes of Taiwan earthquakes with duration magnitude greater than 4 have been routinely determined based on the Richter's $-\log A_o$ values. However, Wang et al. (1989) stressed that the site effects from sediments beneath the station would amplify the short-period signal, thus inflating the $M_L$ value.

From the maximum amplitudes of the displacement seismograms synthesized from the strong-motion accelerograms of 10 events through the technique developed by Kanamori and Jenning (1978), Yeh et al. (1982) obtained an amplitude-distance curve for 0-100 km. Due to small number of data points for epicentral distance greater than 50 km, the deviation of their curve from Richter's increases as the epicentral distance increases. Their amplitude-distance relation is used only by Yeh and his coauthors to establish their catalogues and not used in the routine work to determine local magnitude.

(2) Duration Magnitude

Duration magnitude is a different magnitude estimated from the signal duration (F-P) in seconds by using an empirical formula in the general form:

$$M = a_1 + a_2 \log(F - P) + a_3 \Delta + a_4 h$$

(2)
where \( \Delta \) is the epicentral distance in kilometers, \( h \) is the focal depth in kilometers and \( a_1-a_4 \) are empirical constants. This magnitude was applied to quantify Russian earthquakes first by Bisztricsany (1958) from the duration of surface waves and by Solovev (1965) from the total duration of seismogram. However, they used telemetered seismograms for determining magnitude. Tsumura (1967) determined the duration magnitude from the total duration of oscillation from local earthquakes recorded by Wakayama, Japan microearthquake network. His formulation for determining duration magnitude is still used today in Japan for local earthquakes. Lee et al. (1972) determined an empirical formula for estimating the duration magnitude for California earthquakes in the form:

\[
M_D = -0.87 + 2.00 \log D + 0.0035 \Delta \tag{3}
\]

Lee et al. determined this formula by using 351 central California earthquakes having local magnitude. They found that the \( M_L \) of an earthquake can be estimated by Eq. (3) to within about \( \pm 0.25 \) unit.

Since 1973, Eq. (3) has been introduced to determine the duration magnitude of Taiwan earthquakes by the use of seismograms recorded by the TTSN (Wang, 1989). Since 1988, when a new short-period seismographic network was placed in operation by the CWB, this magnitude scale has also been used by this agency to determine the magnitude for Taiwan earthquakes. The signal duration used by Lee et al. in Eq. (3) was originally defined from the P arrival to the point in the coda where the largest peak-to-peak amplitude on a Geotech model 6585 film viewer (20X magnification) is less than 1 cm. Hence, it is impossible to compare the duration magnitudes determined from different instruments. In other words, earthquake magnitude determined from the total signal duration must be calibrated for each region. The direct use of the duration magnitude formula by Lee et al. to the Taiwan earthquakes is based on the assumption that the geological conditions in California are similar to those in Taiwan. Since the coda waves are caused by the scattering of body waves in the heterogeneous media (Aki, 1969), the coda \( Q \) (\( Q_c \)) is a significant indication to demonstrate the degree of heterogeneity of the media. The \( Q_c \) values for the Taiwan region from Chen et al. (1989), southern California from Mayeda et al. (1991), and central California from Phillips and Aki (1986) are listed in Table 1. It can be seen that the \( Q_c \) values of Taiwan are larger than those of southern California and almost equal to those of central California. Since the formula by Lee et al. (1972) was deduced mainly from the earthquakes in central California, the direct use of their formula to determine duration magnitude for Taiwan earth-

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<th>Frequency (Hz)</th>
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<tr>
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<td>137</td>
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<td>12.0</td>
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quakes seems to be acceptable. According to the magnitude values reported in the Preliminary Determination Epicenters (PDE) by US Geological Survey (USGS), Yiu and Lin (1973) deduced a formula for the duration magnitude for Taiwan earthquakes in the form:

\[ M_D(YL) = 0.632588 + 1.667354 \log D + 0.000582 \Delta \] (4)

But they did not clearly mention which magnitude scale listed in the PDE was used. After an examination of their data set, it is found that their calibration magnitude is the body-wave magnitude. Comparison of Eq. (3) with Eq. (4) shows that the epicentral term is less important in the latter than in the former, and actually can be ignored in the practical calculation by using Eq. (4). However, Yiu and Lin’s formula has not been applied to determine the duration magnitude for Taiwan earthquakes.

According to the coda wave theory, Shin (1986) studied the station correction of Eq. (3) for the TTSN. His revised formula is in the form:

\[ M_D(Shin) = -0.87 + 2.00 \log D + 0.0023 \Delta + R \] (5)

where \( R \) is the station correction and its value is in the range of from -0.01 to 0.45. He also related \( M_D(Shin) \) to \( M_D \) in the form:

\[ M_D(Shin) = 0.955 M_D + 0.16 \] (6)

Essentially, there is only small difference between \( M_D \) and \( M_D(Shin) \).

(3) Body-wave and Surface-wave Magnitudes

From the definition of body-wave and surface-wave magnitudes defined by Gutenberg and Richter in a series of papers, the two magnitude scales were very important for earthquake quantification before 1965. Gutenberg (1945a) defined the surface-wave magnitude in the form:

\[ M_s(GR) = \log A + 1.656 \log \Delta + 1.818 + C \] (7)

In this formula, \( A \) is the vector sum of the maximum amplitudes with period around 20 sec in mm along two horizontal components, \( \Delta \) is the epicentral distance in degree, and \( C \) is the station correction. As only one component amplitude is available, \( A \) is the value of the maximum amplitude multiplied by \( \sqrt{2} \) or 1.4. However, from empirical test, Lienkaemper (1984) showed 1.2 to be a better estimation of the vector sum than 1.4. This formula is mainly appropriate for epicentral distance in the range of from 15° to 130°. For very large earthquakes, the magnitude might be underestimated through Eq. (7). On the other hand, small earthquakes can not be accurately determined by using Eq. (7) due to limited number of data. Lienkaemper also reported that the two horizontal components of the maximum amplitude were not required to be simultaneous by Gutenberg and the periods of the maximum amplitude did not always lie between 18 to 22 sec, actually as low as 12 sec and as high as 23 sec for some cases.

Gutenberg (1945b,c) also defined a body-wave magnitude to classify shallow and deep earthquakes based on P and S waves in the following form:

\[ m_B = \log(A/T) + q(\Delta, h) \] (8)
where $T$ is the period related to the maximum amplitude $A$ and $q(\Delta, h)$ is the correction term associated with epicentral distance ($\Delta$) and focal depth ($h$). Gutenberg (1945a,b) also provided tabulations for the calculation of this term. The maximum amplitude was selected in several ways: (a) the vertical or composite horizontal component of $P$ phase; (b) the vertical or composite-horizontal component of $PP$ phase; and (c) the composite horizontal component of $S$ phase. As only one horizontal component seismogram is available, a value of the maximum amplitude multiplied by $\sqrt{2}$ or 1.4 is taken into account. Before 1950, the intermediate-period instruments were commonly operated, thus the medium-period wave motions were used for the determination of this body-wave magnitude. After careful examination, Abe and Kanamori (1980) stated that in the text of Gutenberg and Richter (1954), for $m_B > 6.9$, the period of $P$ waves used for the determination of magnitude is mainly of from 4 sec to 11 sec with a predominant period of about $7.8 \pm 2.3$ sec for shallow events, $6.4 \pm 1.8$ sec for intermediate-depth events and $5.5 \pm 1.4$ sec for deep events. Gutenberg and Richter (1954) stated that the magnitude for well-observed earthquakes was assigned to the tenth of the unit, with an error less than two tenths, and for the majority of earthquakes, the magnitude was given to the nearest quarter unit. The $M_s$ (GR) and $m_B$ were originally adjusted to coincide near $M = 7$, but were later found to be linearly divergent. Several linear relations were deduced for the two magnitudes by Gutenberg and Richter in a series of papers. Finally, Gutenberg and Richter (1956a) related $M_s$ (GR) to $m_B$ in the form:

$$m_B = 0.63M_s(GR) + 2.5 \tag{9}$$

This formula was applied by them to calculate the $m_B$ from $M_s$ (GR) for the earthquakes whose $m_B$ values could not be determined.

From Gutenberg's original note, Abe and Kanamori (1980) found a sign error in the expression for $m_B$-$M_s$ (GR). They revised this error and deduced a new formula:

$$m_B = 0.57M_s(GR) + 3.0 \tag{10}$$

However, both Eq. (9) and Eq. (10) can not fit the so-called class 'a' data for large earthquakes listed in Geller and Kanamori (1977). But, on the other hand, Gutenberg and Richter (1956a) showed that Eq. (9) fitted the data of $m_B$ vs. $M_s$ (GR) very well. A close examination of Gutenberg and Richter's original data, Abe and Kanamori (1980) stressed that the $m_s$ value (body-wave magnitude calculated from $M_s$ (GR) through Eq. (10)) used in their paper was actually a certain weighted average of $m_B$ and $M_s$ (GR) rather than the real $m_s$. Lienkaemper (1984) stated that $M_{GR}$ used in the text of Gutenberg and Richter was calculated in a form: $M_{GR} = f_1M_s(GR) + f_2m_B$, where $M_B = 1.33(m_B - 1.75)$. For some events, $f_1$ and $f_2$ are $2/3$ and $1/3$, respectively. But actually no single weighting between $M_s$ and $m_B$ to compute $M_{GR}$ held for all events. Hence, Eq. (9) as well as Eq. (10) is not a good fit to the data points of $m_B$ vs. $M_s$ (GR). Besides, Abe and Kanamori (1980) also pointed out that the two equations were determined from the data set which consists of events with $M_s$ (GR) in a limited range of from 6 to 7.5. For large events, Abe and Kanomori (1980) deduced a new conversion formula for $m_B$ and $M_s$ (GR) in the form:

$$m_B = 0.65M_s(GR) + 2.5 \tag{11}$$

Abe (1984) stated that $M_s$ (GR) was approximated by $1.25m_B - 1.75$ for deep and intermediate-depth events.
Since the early 1960's, the World-Wide Standard Seismographic Network (WWSSN) has been installed for monitoring the global earthquakes. The body-wave magnitude and surface-wave magnitude have been determined from the maximum trace body-wave amplitude and surface-wave amplitude, respectively in the seismograms recorded by the WWSSN. The surface-wave magnitude is determined by the so-called "Prague-Moscow formula" by Venek et al. (1962) and denoted as $M_s$:

$$M_s = \log(A/T) + 1.66\log\Delta + 3.3 \quad (12)$$

where $A$ is the peak amplitude, $T$ is the period of the peak amplitude and $\Delta$ is the epicentral distance in degrees. This formula has been accepted by the International Association of Seismology and Physics of Earth's Interior (IASPEI) since 1966 for the determination of surface-wave magnitude of earthquake. In the practical calculation, only the peak amplitude with period of $20\pm2$ sec is used.

Using the data in Gutenberg and Richter's unpublished research notes, Lienkaemper (1984) recomputed $M_s(L)$ through Eq. (7). Comparison of $M_s(GR)$ and $M_s(L)$ leads to two points: (i) single-station magnitudes in the research notes tend to be larger by 0.1 unit of $M_s$ than $M_s(L)$ and (ii) values of $M_s(GR)$ were larger than simple average of all single-station $M_s(L)$ by 0.16 unit of $M_s$ on average. This 0.16 unit excess of $M_s(GR)$ over $M_s(L)$ is close to 0.18 difference between Eq. (7) and Eq. (12) at $T=20$ sec, i.e., $M_s=\log A+1.66\log\Delta+2.0$.

In May, 1968, the United State Coast and Geodetic Survey (USCGS) began publishing in "Earthquake Determination Reports" (EDR) the amplitudes and periods of surface-wave maximum displacements used in the PDE average $M_s$. In September, 1973, PDE operations were transferred to the USGS. Magnitude was computed, until April 1975, with the Prague-Moscow formula using: (i) vector sum of the horizontal components for those maximums with periods $T=18$ to 22 sec, and (ii) for events shallower than 50 km and epicentral distances of $20^\circ$ to $160^\circ$. Beginning May 1975, PDE averages were based on the maximum vertical instead of horizontal component. Although theoretically the ellipticity of Rayleigh waves would make $M_s(\text{Vertical})$ be greater than $M_s(\text{Horizontal})$, the observed differences are negligible (Hunter, 1972; Abe, 1981). The catalog by Gutenberg and Richter (1954) only includes events which occurred before 1954; while the WWSSN was installed after 1960. Hence, it is impossible to compare $M_s(GR)$ and $M_s$ directly. However, in the Rothe's catalog (1969), $M_s(GR)$ was also used. Abe and Kanomori (1980) compared $M_s$ and $M_s(GR)$ from Rothe's catalog and concluded that $M_s(GR)$ is higher than $M_s$ by about 0.1 on the average. Lienkaemper (1984) stressed that $M_s(GR)$ and $M_s$ of PDE differ only slightly for shallow earthquakes ($h<40$ km) and one could treat PDE average $M_s$ as directly comparable to $M_{GR}$ with correction. He also mentioned that adding 0.06 to $M_s$ values published in Abe (1981) for events between 1910 to 1952, $h<40$ km would adjust them to a scale compatible with PDE $M_s$. Since the installation of the WWSSN in the early 1960's, the body-wave magnitude has been determined almost exclusively from the vertical component of the $P$ wave ground motions at a period of approximately 1 sec through Eq. (8) and represented by $m_B$. The difference between $m_B$ and $m_B$ has been studied by numerous authors. Guyton (1964) stated that the $m_B$ values for a single earthquake, determined from body waves at different seismographic stations, commonly vary by 0.5 or more, despite corrections for differences in epicentral distance among the stations. This variation, which is related to the differences in amplitudes of a factor of 3 or more, is generally due to azimuthal, instrumental and geological differences among the stations. Romney (1964) and Geller and Kanamori (1977) reported that the $m_B$ values are about 0.3-0.6 units higher than the $m_b$ values. Abe and Kanamori (1980) expressed that $m_B$ is systematically larger than $m_b$ by about 1.3 on the average for events with $m_B>7$. 


For $5.5 < m_B < 7.8$, Abe (1981) stated that $m_b$ is lower than $m_B$ by about 0.4-1.1 units. He also deduced a relation for the two magnitudes in the form:

$$m_B = 1.5m_b - 2.2$$

(13)

(4) Hsu’s Magnitude

In order to determine the magnitude for Taiwan earthquakes, Hsu (1971) corrected the surface-wave magnitude, measured from the seismograms recorded by the WWSSN, to the maximum trace amplitude ($A$) and epicentral distance ($\Delta$) recorded by the displacement-type seismographs of the old network of the CWB. Since the number of earthquakes having $M_s$ value was very small before 1970, he had to calculate the $M_s$ values from the $m_B$ values. However, the conversion formula between the two magnitudes for Taiwan earthquakes was not given at that time. He had to use a $M_s$-$m_b$ relation:

$$M_s = 0.76m_b + 1.58$$

(14)

obtained by Ichikawa (1966) for Japanese earthquakes for conversion. His formulae for estimating the magnitude actually vary at different stations. But for the practical computation, he suggested an average formula:

$$M_H = \log A + 1.09\log \Delta + 0.50$$

(15)

It is noted that this relation is applied to determine the $M_H$ values at several stations, and then the average $M_H$ value is calculated from the given $M_H$ values. Hsu used this magnitude scale to quantify Taiwan earthquakes before 1978. The $M_s$-$m_b$ conversion formula for Japanese earthquakes is different from that for Taiwan earthquakes:

$$M_s = 1.356m_b - 1.736$$

(16)

by Wang (1985). As shown in Figure 1, for $m_b > 5.56$, $M_s$(Taiwan) is higher than $M_s$(Japan) and vice verse for $m_b < 5.56$. Hence, the $M_H$ might be overestimated for $m_b < 5.56$ and underestimated for $m_b > 5.56$.

**Fig. 1.** Figure shows the $M_s$-$m_b$ relations for Taiwan earthquakes (in solid line) and Japanese earthquakes (in dashed line).
(5) JMA Magnitude

The magnitudes of earthquakes in Japan and some larger earthquakes in Taiwan are routinely determined by the Japanese Meteorological Agency (JMA, formerly Central Meteorological Observatory) by using the formula obtained by Tsuboi (1951):

$$M_J = \log A + 1.731 \log \Delta - 0.83$$

(17)

where $A$ is either the larger value of the maximum amplitudes along two horizontal components or the composite value of the two maximum amplitudes in $\mu$m and $\Delta$ is the epicentral distance in km. This magnitude was denoted as $M_{IJ}$ in Wang and Miyamura (1990) and Wang et al. (1990). Hayashi and Abe (1984) reported that the average period of wave motions used for determining $M_J$ is about 3 sec and this magnitude agrees very well with $M_s$. However, $M_J$ deviates very systematically from $M_s$ as $M_s$ decreases, and $M_J$ is overestimated by as much as 0.6 at $M_s=4$.

(6) Kawasumi's Intensity Magnitude

Kawasumi (1943) defined a magnitude $M_I$ (denoted by $M_K$ in his papers) based on the intensity value at an epicentral distance of 100 km. The intensity scale is the Japanese scale in 8 degrees from 0 to VII, which has been used in Taiwan by combining VI and VII to be VI. The formula for the conversion of intensity of degree I and magnitude $M_I$ as the epicentral distance ($\Delta$) is not equal to 100 km is in the form:

$$I = M_I + 2\ln(100/\Delta) - 0.00183(\Delta - 100)$$

(18)

and

$$I = M_I + 2\log(r_o/r) - 0.01668(r - r_o)$$

(19)

where $\Delta$ = epicentral distance; $r$ = hypocentral distance; and $r_o$ = hypocentral distance at $\Delta=100$ km. Later, Kawasumi (1951) related $M_I$ to $M_L$ in the following form:

$$M_L = 4.85 + 0.5M_I$$

(20)

Wang et al. (1990) expressed that the correlation between $M_I$ with other magnitudes is not good enough. Thus, they proposed that the $M_I$ might be not an appropriate magnitude to quantify Taiwan earthquakes.

(7) Moment Magnitude

The seismic moment $M_o=\mu Au$, where $\mu$ is the shear modulus, $A$ is the fault area and $u$ is the spatial average slip on the fault during the earthquake occurrence, was first applied by Aki (1966) to quantify earthquake. The seismic moment can be related to the energy release in earthquakes. Aki (1966, 1967) showed that the amplitude of very long period waves is proportional to $M_o$ and Ben-Menahem et al. (1969) also stated that the far-field static-strain field is also proportional to $M_o$. Besides, because $M_o$ does not saturate, it is a good parameter to represent the size of great earthquakes and has been applied to define moment magnitude by Kanamori (1977) and Hanks and Kanamori (1979). Kanamori (1977)
related the seismic energy ($E_s$) given by $M_o/(2 \times 10^4)$ to a moment magnitude using the formula by Gutenberg and Richter (1956b):

$$\log E_s = 1.5M_s + 11.8$$  \hspace{1cm} (21)

The moment magnitude ($M_w$) is defined as

$$M_w = (2/3)\log M_o - 10.7$$  \hspace{1cm} (22)

under an assumption that stress drop is constant. In Eq. (22), $M_o$ is in the unit of dyne-cm. Hanks and Kanamori (1979) stated that Eq. (22) is uniformly valid for $3 < M_L < 7$, $5 < M_s < 7.5$ and $M_w > 7.5$. The $M_o$ values for larger events can be found in the EDR. According to the method proposed by Bolt and Herriz (1983), Li and Chiu (1989) estimated the seismic moment of Taiwan earthquakes from the simulated Wood-Anderson seismograms. Their resultant formula is in the form:

$$\log M_o(LC) = (16.74 \pm 0.20) + (1.22 \pm 0.14)\log(C \times D \times \Delta)$$  \hspace{1cm} (23)

where $C$ is the peak-to-peak amplitude, $D$ is the duration between the S-arrival and the onset of the signal with amplitude of $C/d$ and $\Delta$ is the epicentral distance. They stated that the optimum estimation for seismic moment can be obtained as $d=2$.

### 3. RELATIONS BETWEEN MAGNITUDE SCALES

The relations between magnitudes obtained by numerous authors will be described as follows. It is noted that the data points of $m_b$-$M_H$ cannot be described by a single regression equation due to high dispersion (Wang and Miyamura, 1990), thus will not be discussed further. Basically six groups of relations are discussed.

**1. Relations of $M_L$ vs. $M_D$, $M_L$ vs. $M_H$ and $M_L$ vs. $m_b$**

Three relations of $M_L$ vs. $M_D$ were studied by three groups of authors. They are

$$M_L = 0.33 + 1.04M_D \pm 0.23$$  \hspace{1cm} (24)

by Liaw and Tsai (1981);

$$M_L(Yeh) = 1.10 + 0.93M_D \pm 0.30$$  \hspace{1cm} (25)

by Yeh et al. (1982); and

$$M_D = (0.187 \pm 0.373) + (0.862 \pm 0.066)M_L$$  \hspace{1cm} (26)

by Wang et al. (1989). Since $M_L$ determined by Yeh et al. (1982) was based on the $-\log A_o$ values obtained by themself and is denoted by $M_L(Yeh)$. The three equations are plotted in Figure 2. It is obvious that the three equations are close to one another despite the
Fig. 2. Figure shows the $M_L$-$M_D$ relations from Liaw and Tsai (1981) in dashed line, Yeh et al. (1982) in dotted line and Wang et al. (1989) in solid line.

use of different data sets to determine the equations. Hsu’s catalog in Hsu (1971, 1980, and 1985) contains the most complete instrumentally-determined seismic data during 1900-1978. It is necessary to compare local magnitude $M_L$, which has been used since 1973, with $M_H$ before the establishment of a complete catalog for Taiwan earthquakes. Yeh et al. (1982) first related $M_H$ to their local magnitude $M_L(Yeh)$ in the form:

$$M_L(Yeh) = 2.63 + 0.56M_H \pm 0.27 \quad (27)$$

A relation between the two magnitudes was determined by Yeh and Hsu (1985) in the following form:

$$M_L(Yeh) = 1.04 + 0.94M_H \pm 0.28 \quad (28)$$

Cheng and Yeh (1989) obtained a slightly different form for the relation between the two magnitudes:

$$M_L(Yeh) = 1.42 + 0.80M_H \pm 0.27 \quad (29)$$

The three equations are shown in Figure 3. It is evident that for $M_H > 6$, the $M_L$ values

Fig. 3. Figure shows the $M_L$-$M_H$ relations from Yeh et al. (1982) in dashed line, Yeh and Hsu (1985) in dotted line and Cheng and Yeh (1989) in solid line.

determined from Eq. (28) are larger than those from Eqs. (27) and (29) by 0.5 unit. It is interesting and necessary to compare $m_b$ and $M_L$. Both of them are determined from the
peak amplitudes of around 1 sec: \( m_b \) is according to the telemetered P waves, while \( M_L \) is based on the local S waves or Lg waves. Three relations between the two magnitudes have been determined:

\[
m_b = 0.27 + 0.85M_L \pm 0.60 \tag{30}
\]

by Shin (1986);

\[
M_L = (-0.604 \pm 0.485) + (1.268 \pm 0.094)m_b \tag{31}
\]

by Wang et al. (1989); and

\[
M_L = 1.94 + 0.75m_b \tag{32}
\]

by Cheng and Yeh (1989). The three regression equations are shown in Figure 4.

Fig. 4. Figure shows the \( M_L-m_b \) relations from Shin (1986) in dashed line, Cheng and Yeh (1989) in dotted line and Wang et al. (1989) in solid line.

Essentially, Eqs. (30) and (31) are close to each other, while Eq. (32) remarkably deviates from the other two. For \( m_b < 5 \), the \( M_L \) values from Eq. (32) are smaller than those from Eqs. (30) and (31), but vice versa for \( m_b > 5 \).

(2) Relations of \( M_D \) vs. \( m_b \) and \( M_D \) vs. \( M_s \)

Wang and Chiang (1987) compared \( M_D \) with \( m_b \) and \( M_s \) for shallow earthquakes with focal depth less than 40 km and deep ones with focal larger than 40 km. The data points for \( M_D \) vs. \( m_b \) are quite dispersive and most events have \( m_b \) values in a small range of from 4.8 to 5.5. The \( M_D \)-\( m_b \) relation for earthquakes with focal depth greater than zero is:

\[
M_D = (-1.193 \pm 0.459) + (1.211 \pm 0.097)m_b \tag{33}
\]

Although the number of data points of \( M_D \) vs. \( M_s \) is small, their relation was determined by Wang and Chiang (1987) in the form:

\[
M_D = (3.442 \pm 0.632) + (0.374 \pm 0.106)M_s \tag{34}
\]

Shin (1986) related \( m_b \) to \( M_D(Shin) \) in the form:

\[
m_b = 0.3 + 0.92M_D(Shin) \pm 0.5 \tag{35}
\]
Eqs. (33) and (35) are similar as shown in Figure 5.

![Figure 5](image)

**Fig. 5.** Figure shows the $M_D$-$m_b$ relations from Shin (1986) in dashed line and Wang and Chiang (1987) in solid line.

### (3) Relations of $M_H$ and other magnitudes

Since the relation between $M_H$ and $M_L$ was given in (1) of this section, only the relations of $M_H$ vs. $M_s(GR)$, $M_H$ vs. $M_s$, $M_H$ vs. $m_b$ and $M_H$ vs. $M_J$ are presented here. It is noted that the $M_H$ is determined from the seismograms recorded at the local stations, while the other four magnitude scales are determined from the seismograms recorded at the stations outside Taiwan.

The relations of $M_s(GR)$ vs. $M_H$ and $M_J$ vs. $M_H$ given by Wang et al. (1990) are:

$$M_s(GR) = (1.12 \pm 0.59) + (0.85 \pm 0.08)M_H$$

(36)

and

$$M_J = (1.26 \pm 0.55) + (0.82 \pm 0.08)M_H$$

(37)

The two regression equations are very similar. As the previous mention, the $M_H$ was originally defined based on the surface-wave magnitude $M_s$. A comparison between the two magnitudes is significant. Figure 6 shows the data points of $M_s$ vs. $M_H$. The regression equation for the data points is in the form:

$$M_s = (-0.95 \pm 0.31) + (1.15 \pm 0.05)M_H$$

(38)

![Figure 6](image)

**Fig. 6.** Figure shows the data points (in open circle) of $M_s$ vs. $M_H$, the related regression equation (in solid line) and the bisection line (in dashed line).
The bisection line (denoted by dashed line in Figure 6) is very similar to the regression line (in solid line), thus implying the equality of the two magnitudes for Taiwan earthquakes. Although the $M_H$ was determined from local seismic data, it is like the surface-wave magnitude $M_s$. However, from Eq. (38), as $M_H > 6.3$, $M_H < M_s$ and as $M_H < 6.3$, $M_H > M_s$.

It is also interesting to compare $M_H$ with $m_b$ because the determination of $M_H$ was actually originally from $m_b$ through a conversion formula of $M_s$ and $m_b$. Figure 7 shows

Fig. 7. Figure shows the data points (in open circle) of $m_b$ vs. $M_H$, the related regression equation (in solid line) and the bisection line (in dashed line).

the data points of $m_b$ vs. $M_H$. It is obvious that almost all data points are below the bisection line. The regression equation to fit the data points is in the form:

$$m_b = (1.96 \pm 0.24) + (0.59 \pm 0.00) M_H$$

Although Wang et al. (1990) suggested that $M_I$ is not an appropriate magnitude to quantify Taiwan earthquakes, for the purpose of reference, the relation between $M_I$ and $M_H$ for $M_I < 8$ is presented as:

$$M_I = (3.700 \pm 0.512) + (0.409 \pm 0.081) M_H$$

(4) Relation of $M_s(GR)$ vs. $m_B$

$M_s(GR)$ and $m_B$ are two magnitudes scales used by Gutenberg and Richter to quantify earthquakes before 1954. Their relation for Taiwan earthquakes is in the form:

$$m_B = (0.17 \pm 0.80) + (0.96 \pm 0.11) M_s(GR)$$

by Wang and Miyamura (1990). This equation is different from that obtained by Gutenberg and Richter (1954):

$$m_B = 2.5 + 0.63 M_s(GR)$$

for global earthquakes.

(5) Relations of $M_J$ vs. $M_s(GR)$ and $M_J$ vs. $m_B$

The relation between $M_J$ and $M_s(GR)$ is in the form:

$$M_J = (0.25 \pm 0.34) + (0.96 \pm 0.05) M_s(GR)$$

(43)
by Wang and Miyamura (1990). Although the $M_s$(GR) values were determined by taking the mean of the maximum amplitudes of wave motions from various raypaths worldwide, while the $M_J$ values were determined only from those passing through the region between Taiwan and Japan, high correction of the two regression equations implicates the equality of the two magnitudes for Taiwan earthquakes.

The relation between $M_J$ and $m_B$ is

$$M_J = (-2.68 \pm 1.38) + (1.38 \pm 0.19)m_B$$


(6) Relations of $M_o$ vs. $M_s$, $M_o$ vs. $m_b$ and $M_o$ vs. $M_L$

Since the $M_o$ values were not determined for earthquakes which occurred before 1970, there is an attempt to estimate the $M_o$ values for such events through a simple method by the conversion formula between $M_o$ and magnitude. For Taiwan earthquakes, Wang (1985) related $M_o$ to $M_s$ in the form:

$$\log M_o = 1.20 M_s + 17.83$$

and to $m_b$ in the form:

$$\log M_o = 1.90 m_b + 14.19$$

The two regression equations for Taiwan earthquakes agree closely with the average seismic moment-magnitude relations for the Pacific plate margin earthquakes obtained by Nuttli (1983). But the $M_s$-$m_b$ relation for Taiwan earthquakes is different from that for the Pacific plate margin ones by Nuttli (1983). Seismic moment calculated from very long-period surface waves is associated with static property of the fault; while the $M_s$ determined from surface waves with period of about 20 sec and the $m_b$ determined from body waves with period of about 1 sec are both related to kinematic rupture on the fault. Given results might show the tectonics in the Taiwan region are similar to that in the whole Pacific plate margin but the rupture process of earthquake in the former might be different from the average one in the latter. The relation between $M_o$ and $M_L$ is in the form:

$$\log M_o = (14.571 \pm 1.683) + (1.598 \pm 0.236)M_L$$

by Wang et al. (1989). The $M_o$(LC) determined the formula by Li and Chiu (1989) is related to the $M_L$ in the form:

$$\log M_o(LC) = (19.043 \pm 0.533) + (0.914 \pm 0.035)M_L$$

Eqs. (47) and (48) are shown in Figure 8. It is obvious that the two equations do not agree with each other. The $M_o$ values applied to determine Eq. (47) can be considered as the standard ones because they were estimated from very long-period surface waves. Hence the difference of Eqs. (47) and (48) might show that the $M_o$(LC) values were overestimated for $M_L<6.5$ and underestimated for $M_L<6.5$. 
Fig. 8. Figure shows the $\log M_o$-$M_L$ relations from Li and Chiu (1989) in dashed line and Wang et al. (1989) in solid line.

The selection of the "d" value in Eq. (23) is questionable. The optimum value chosen by Bolt and Herraiz (1983) was the time between the $S$ (with amplitude C) onset and the point having an amplitude $c/C=1/3$, while Li and Chiu's result is $c/C=1/2$. A physically reasonable interpretation about the d value is needed before the use of the Bolt and Herraiz's technique (1977) to determine $M_o$ from local seismograms.

4. CONCLUSIONS

From the above discussion, several points can be derived as follows:
1. The $M_L$-$M_D$ relations obtained by three groups of authors are similar.
2. The three $M_L$-$M_H$ relations obtained by Yeh and his coauthors are essentially the same.
3. The $M_L$-$m_b$ relations obtained by Shin (1986) and Wang et al. (1989) are close to each other even their data sets are different. But they are remarkably different from that obtained by Cheng and Yeh (1989).
4. The $M_D$-$m_b$ relations obtained by Shin (1986) and Wang and Chiang (1987) are almost the same.
5. Although the formula to determine the $M_H$ by Hsu (1971) was originally defined based on a Japanese $M_s$-$m_b$ conversion relation and determined from local seismograms, the correction between $M_H$ and $M_s$(GR) as well as $M_H$ and $M_s$ for Taiwan earthquakes is good enough. Consequently, although Hsu’s magnitude was determined from local seismograms, it is like a surface-wave magnitude in the practical use.
6. The $\log M_o$-$M_L$ relations obtained by Li and Chiu (1989) and Wang et al. (1989) are quite different. For $M_L>6.65$, $M_o$(LC)$<M_o$(EDR) and for $M_L<6.65$, $M_o$(LC)$>M_o$(EDR). In other words, $M_o$(LC) might be overestimated for $M_L<6.5$ and underestimated for $M_L>6.5$.

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REFERENCES


Yeh, Y.T. and P.S. Hsu, 1985: Catalog of Earthquakes in Taiwan from 1644 to 1984, unpublished manuscript.


台灣地震的各種規模及其等間關係式之回顧

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摘要

本文回顧並說明自1900年以來用於表示台灣地震大小之各種規模，如$M_L$, $M_D$, $M_s$(GR), $m_B$, $M_s$, $m_b$, $M_H$, $M_J$和$M_I$。同時這些規模間之關係式亦一併在文中討論。